WORKSHEET: MEAN VALUE THEOREM; L'HOPITAL'S RULE 2

Names and student IDs: _____

The first problem is about the Mean Value Theorem. The second is on a case in which L'Hopital's Rule appears to apply, but for which it does not actually work. We already did an example like this one in the lecture of Monday 2 June, using methods like those on the worksheet of Tuesday 3 June. If we don't get to this problem in class today (likely), do it at home.

1. Let f be a differentiable function defined on the open interval $(0, 8)$. Suppose $f(1) = 10$ and $f'(x) \geq 5$ for all x in $(1, 4)$. What can you say about $f(4)$? Your answer should have one of the forms

$$f(4) \geq ____, \qquad f(4) > ____, \qquad f(4) \leq ____, \qquad \text{or} \qquad f(4) < ____.$$

Reminders on L'Hopital's Rule.

Do NOT confuse L'Hopital's Rule with the quotient rule!
Before using L'Hopital's Rule, you must check that its hypotheses are satisfied!

A special case: **if** $\lim_{x\to\infty} f(x) = \pm\infty$ **and** $\lim_{x\to\infty} g(x) = \pm\infty$**,** and if $\lim_{x\to\infty} \frac{f'(x)}{g'(x)}$ exists, then

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}.$$

(Generally, one must have a fraction with an **indeterminate form**, such as "$\frac{0}{0}$" or "$\frac{\pm\infty}{\pm\infty}$".)

2. Consider $\lim_{x\to\infty} \frac{2x}{x + \sin(3x)}$. Set $f(x) = 2x$ and $g(x) = x + \sin(3x)$. Reminders for this problem: $\lim_{x\to\infty} \cos(3x)$ does not exist (the function oscillates between -1 and 1) and, by the Squeeze Theorem, $\lim_{x\to\infty} \frac{\sin(3x)}{x} = 0$.

(a) Find $\lim_{x\to\infty} f(x)$.

(b) Find $\lim_{x\to\infty} g(x)$.

(c) Does $\lim_{x\to\infty} \frac{2x}{x + \sin(3x)}$ have an indeterminate form?

(d) Can we try L'Hopital's Rule on $\lim_{x\to\infty} \frac{2x}{x + \sin(3x)}$? Why?

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(e) What happens if we try L'Hopital's Rule on $\lim_{x\to\infty} \dfrac{2x}{x+\sin(3x)}$?

(f) Use some other method to find $\lim_{x\to\infty} \dfrac{2x}{x+\sin(3x)}$. Hint: Multiply the numerator and denominator by $\frac{1}{x}$. We used this method on a similar problem in class Monday.