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# MATH 251 (PHILLIPS) MIDTERM ZERO (SAMPLE 1)

Turn in this version of the sample Midterm Zero as homework Tuesday 7 January 2025. As on the real version:

(1) No partial credit. This mans getting no credit even for minor mistakes, even notation, like using the wrong variable in an answer.
(2) Fully correct notation is required. There are specific notation warnings on some problems; these will **not** appear on the real Midterm Zero.
(3) Work is not required, and will not be graded.
(4) All answers must be simplified as much as possible.
(5) Write all answers in the spaces provided at the right.

The real Midterm Zero will allow no books, notes, calculators, or other electronic devices.

**Problems:**

1. Write as a single fraction: $\dfrac{2}{x-1} - \dfrac{1}{x+2}$

Solution:

$$\frac{2}{x-1} - \frac{1}{x+2} = \frac{2(x+2)-(x-1)}{(x-1)(x+2)} = \frac{2x+4-x+1}{(x-1)(x+2)} = \frac{x+5}{(x-1)(x+2)}.$$

**Notation reminder for Problem 1** (will **not** be repeated on the real Midterm Zero): to avoid ambiguity, all fraction lines must be exactly horizontal and cover the entire numerator and denominator, unless enough parentheses are used. Thus, for the problem $\dfrac{1}{x-17} - \dfrac{1}{x-1}$, the following answers use correct notation and will get full credit:

$$\frac{16}{(x-17)(x-1)} \qquad \frac{16}{x^2-18x+17} \qquad 16/\big[(x-17)(x-1)\big] \qquad 16/(x^2-18x+17).$$

The following answers (crossed out in red in the online version of this file) will get **zero** credit because of notation errors:

$$\cancel{16/x^2-18x+17} \qquad \cancel{16/(x-17)(x-1)} \qquad \cancel{\frac{16}{x^2-18x+17}}$$

$$\cancel{\frac{16}{(x-17)(x-1)}} \qquad \cancel{\frac{16}{x^2-18x+17}} \qquad \cancel{\frac{16}{(x-17)(x-1)}}$$

For more, see Section 3 of the posted file on notation.

2. Simplify the following expression as much as possible. If no simplification is possible, write "not possible": $\dfrac{4\cos(4y)}{4\cos(4y)+6}$

Solution:

$$\frac{4\cos(4y)}{4\cos(4y)+6} = \frac{2[2\cos(4y)]}{2[2\cos(4y)+3]} = \frac{2\cos(4y)}{2\cos(4y)+3}.$$

The last expression can't be further simplified.

3. Let $h(x) = 2 - x^2$. Evaluate the expression $h(2) - h(x+2)$, and simplify it as much as possible.

Solution:

$h(2) - h(x+2) = 2 - 2^2 - (2 - (x+2)^2) = 2 - 4 - 2 + (x+2)^2 = -4 + x^2 + 4x + 4 = x^2 + 4x.$

4. Find all real solutions to the equation $1 + 3x^{-1} = 4x^{-2}$. If no real solution exists, write "no solution".

Solution:

$$1 + 3x^{-1} = 4x^{-2}$$
$$1 + 3x^{-1} - 4x^{-2} = 0$$

Multiply through by $x^2$:

$$x^2 + 3x - 4 = 0$$
$$(x-1)(x+4) = 0$$
$$x = 1 \qquad \text{or} \qquad x = -4.$$

Since there is no partial credit, no credit is given for only one of the two solutions. Note that both answers actually are solutions to the original equation, that is, that multiplying both sides by $x^2$ at the first step did not introduce any extraneous solutions.

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One can also solve this problem by treating it as a quadratic equation in $x^{-1}$.

5. Find all real solutions to the equation $8(2 - x^{-3}) = 16$. If no real solution exists, write "no solution".

Solution:

$$16 - 8x^{-3} = 16$$
$$-8x^{-3} = 0$$

Multiply both sides by $x^3$ to get $-8 = 0$. Therefore there are no solutions. (Alternatively, it is obvious that $-8x^{-3}$ can never be zero.)

6. Multiply out: $(x+2)(x^2 - x + 1)$.

Solution:

$$(x+2)(x^2 - x + 1) = x^3 + 2x^2 - x^2 - 2x + x + 2 = x^3 + x^2 - x + 2.$$

7. Find all real numbers $a$ such that $|a| = -a$.

**Notation reminder for Problem 7** (will **not** be repeated on the real Midterm Zero): Use the correct variable! See Section 7 of the posted file on notation.

Solution: $|a| = -a$ if and only if $a \leq 0$.

8. Find all real solutions to the equation $2e^{2x+2} - 7 = 11$. If no real solution exists, write "no solution".

Solution:

$$\begin{aligned} 2e^{2x+2} - 7 &= 11 \\ 2e^{2x+2} &= 18 \\ e^{2x+2} &= 9 \\ 2x + 2 &= \ln(9) \\ x = \tfrac{1}{2}(\ln(9) - 2) &= \tfrac{1}{2}\ln(9) - 1 = \ln(3) - 1 \end{aligned}$$

The final simplification is *necessary*.

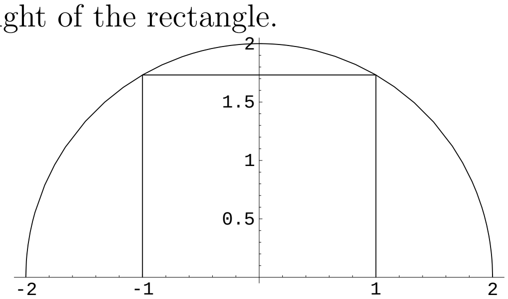
9. Simplify completely (for $x \neq 0$): $\left( \dfrac{2\sqrt{3}}{3x^{-1}} \right)^2$

Solution:

$$\left( \frac{2\sqrt{3}}{3x^{-1}} \right)^2 = \frac{2^2(\sqrt{3})^2}{3^2x^{-2}} = \frac{4 \cdot 3}{3^2x^{-2}} = \frac{4x^2}{3}.$$

If you want, you can rewrite the answer as $\frac{4}{3}x^2$, but that is not necessary.

10. The graph of $f(x) = \sqrt{4 - x^2}$ is the semicircle shown below. Give the exact value of the height of the rectangle.




Solution: The height of the rectangle is $f(1) = \sqrt{4 - 1^2} = \sqrt{3}$. (You can also get it by calculating $f(-1)$. This gives the same answer.)