MATH 251 (PHILLIPS) MIDTERM 0 VERSION 1, 9:00 am F 10 January 2025

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INSTRUCTIONS: No books, notes, or calculators are permitted on this test. All answers must be simplified as much as possible. Write all answers in the spaces provided at the right. Do scratchwork on the back or on blank paper provided for this purpose. *No partial credit.* Time: 20 minutes.

1. Find all real solutions to the equation $9z^{-4} = 0$. If no real solution exists, write "no solution".

Solution: Multiply both sides by z^4 to get $9 = 0$. Therefore there are no solutions. (Alternatively, write $9z^{-4} = 9/z^4$, which can obviously never be zero.)

2. Write as a single fraction, and simplify as much as possible: $\dfrac{2}{x-6} - \dfrac{1}{x+5}$

Solution:

$$\frac{2}{x-6} - \frac{1}{x+5} = \frac{2(x+5)-(x-6)}{(x-6)(x+5)} = \frac{2x+10-x+6}{(x-6)(x+5)} = \frac{x+16}{(x-6)(x+5)}.$$

3. Let $f(x) = 3 - x$. Evaluate the expression $f(4) - f(x-2)$, and simplify it as much as possible.

Solution:

$$f(4) - f(x-2) = 3 - 4 - \big(3 - (x-2)\big) = 3 - 4 - 3 + x - 2 = x - 6.$$

4. Find all real solutions to the equation $2e^{11x} - 9 = 13$. If no real solution exists, write "no solution".

Solution:

$$\begin{aligned} 2e^{11x} - 9 &= 13 \\ 2e^{11x} &= 22 \\ e^{11x} &= 11 \\ 11x &= \ln(11) \\ x &= \tfrac{1}{11}\ln(11). \end{aligned}$$

Note that $\frac{1}{11}\ln(11)$ can't be further simplified.

5. Find the domain of the function $f(x) = \sqrt{-x}$.

Solution: $\sqrt{-x}$ is defined if and only if $-x \geq 0$, which happens if and only if $x \leq 0$. Therefore the domain is $(-\infty, 0]$.

6. Find all real solutions to the equation $\left(\dfrac{x}{6}\right)(5-x) = 1$. If no real solution exists, write "no solution".

Solution:

$$\begin{aligned} \left(\frac{x}{6}\right)(5-x) &= 1 \\ x(5-x) &= 6 \\ 5x - x^2 &= 6 \\ 0 = x^2 - 5x + 6 &= (x-2)(x-3) \\ x = 2 \qquad &\text{or} \qquad x = 3. \end{aligned}$$

Since there is no partial credit, no credit is given for only one of the two solutions.

7. Simplify the following expression as much as possible. If no simplification is possible, write "not possible": $\dfrac{w^3+8w}{w^3+2w}$

Solution:

$$\frac{w^3+8w}{w^3+2w} = \frac{w(w^2+8)}{w(w^2+2)} = \frac{w^2+8}{w^2+2}.$$

The last expression can't be further simplified.

8. Multiply out: $(t+1)(t^2-3t-1)$.

Solution:

$$(t+1)(t^2-3t-1) = t^3+t^2-3t^2-3t-t-1 = t^3-2t^2-4t-1.$$

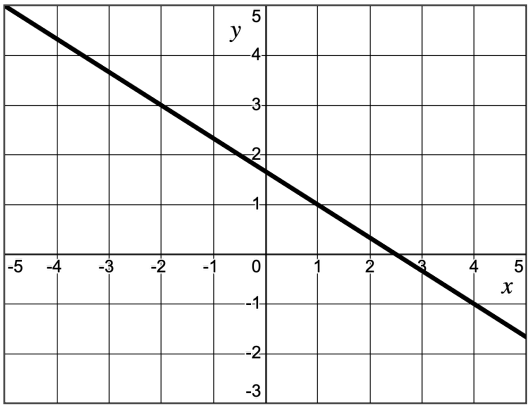
9. Simplify completely (for $c>0$): $\left(\dfrac{c^3\sqrt{5}}{5\sqrt{c}}\right)^2$

Solution:

$$\left(\frac{c^3\sqrt{5}}{5\sqrt{c}}\right)^2 = \frac{\left(c^3\sqrt{5}\right)^2}{\left(5\sqrt{c}\right)^2} = \frac{5c^6}{5^2c} = \frac{c^5}{5}.$$

If you like, you can rewrite this as $\frac{1}{5}c^5$, but that is not necessary.

10. Determine the exact value of the **slope** of the line in the graph below.



Solution: You can tell by reading the graph that the points $(x_1, y_1) = (1, 1)$ and $(x_2, y_2) = (-2, 3)$ are on the line. Therefore the slope is

$$\frac{y_1-y_2}{x_1-x_2} = \frac{1-3}{1-(-2)} = \frac{-2}{3} = -\frac{2}{3}.$$

Another approach is to simply observe from the graph that the line goes down two units for each three units to the right.