MATH 251 (PHILLIPS) MIDTERM 0 VERSION 1, 11:00 am F 10 January 2025

NAME: <u>SOLUTIONS</u> Student id: $\underline{\pi\pi\pi\text{-}\pi\pi\text{-}\pi\pi\pi\pi}$

INSTRUCTIONS: No books, notes, or calculators are permitted on this test. All answers must be simplified as much as possible. Write all answers in the spaces provided at the right. Do scratchwork on the back or on blank paper provided for this purpose. *No partial credit.* Time: 20 minutes.

1. Write as a single fraction, and simplify as much as possible: $\dfrac{3}{a-2} - \dfrac{1}{a-5}$

Solution:

$$\frac{3}{a-2} - \frac{1}{a-5} = \frac{3(a-5)-(a-2)}{(a-2)(a-5)} = \frac{3a-15-a+2}{(a-2)(a-5)} = \frac{2a-13}{(a-2)(a-5)}.$$

2. Multiply out: $(a+1)(a^2-3a+1)$.

Solution:

$$(a+1)(a^2-3a+1) = a^3+a^2-3a^2-3a+a+1 = a^3-2a^2-2a+1.$$

3. Let $f(x) = 5-x$. Evaluate the expression $f(x+3)-f(3x)$, and simplify it as much as possible.

Solution:

$$f(x+3)-f(3x) = 5-(x+3)-(5-3x) = 5-x-3-5+3x = 2x-3.$$

4. Simplify the following expression as much as possible. If no simplification is possible, write "not possible": $\dfrac{6x^2+3}{6x^2+12}$

Solution:

$$\frac{6x^2+3}{6x^2+12} = \frac{3(2x^2+1)}{3(2x^2+4)} = \frac{2x^2+1}{2x^2+4}.$$

The last expression can't be further simplified, although it can be rewritten as $\dfrac{2x^2+1}{2(x^2+2)}$.

5. Find all real solutions to the equation $\ln(5x+6) = 5$. If no real solution exists, write "no solution".

Solution:

$$\begin{aligned} \ln(5x+6) &= 5 \\ 5x+6 &= e^5 \\ 5x &= e^5-6 \\ x &= \frac{e^5-6}{5}. \end{aligned}$$

The answer can be rewritten as $x = \frac{1}{5}e^5 - \frac{6}{5}$, but this is not necessary. However, it can't be further simplified.

6. Find all real solutions to the equation $\dfrac{1}{9x^2} = 0$. If no real solution exists, write "no solution".

Solution: Multiply both sides by $9x^2$ to get $1 = 0$. Therefore there are no solutions. (Alternatively, it is obvious that $\dfrac{1}{9x^2}$ can never be zero.)

7. Find all real solutions to the equation $\left(\frac{x}{3}\right)(x+2) = 1$. If no real solution exists, write "no solution".

Solution:

$$\begin{aligned} \left(\frac{x}{3}\right)(x+2) &= 1 \\ x(x+2) &= 3 \\ x^2 + 2x &= 3 \\ 0 = x^2 + 2x - 3 &= (x-1)(x+3) \\ x = 1 \qquad \text{or} &\qquad x = -3. \end{aligned}$$

Since there is no partial credit, no credit is given for only one of the two solutions.

8. Find the domain of the function $g(x) = \dfrac{1}{\sqrt{-x}}$.

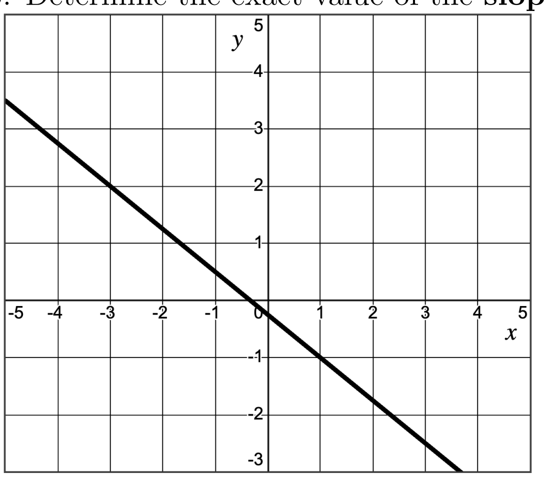
Solution: x is in the domain if and only if $-x \geq 0$ (so that $\sqrt{-x}$ is defined) and $\sqrt{-x} \neq 0$ (so that $\dfrac{1}{\sqrt{-x}}$ is also defined). The first condition requires $x \leq 0$, and the second rules out $x = 0$, so the answer is all real x with $x < 0$.

9. Simplify completely (for $y > 0$): $\left(\dfrac{2y^{3/2}}{\sqrt[3]{2}\,y}\right)^3$

Solution:

$$\left(\frac{2y^{3/2}}{\sqrt[3]{2}\,y}\right)^3 = \frac{2^3(y^{3/2})^3}{\left(\sqrt[3]{2}\right)^3 y^3} = \frac{2^3 y^{9/2}}{2y^3} = 4y^{3/2}.$$

10. Determine the exact value of the **slope** of the line in the graph below.



Solution: You can tell by reading the graph that the points $(x_1, y_1) = (-3, 2)$ and $(x_2, y_2) = (1, -1)$ are on the line. Therefore the slope is

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{2 - (-1)}{-3 - 1} = \frac{3}{-4} = -\frac{3}{4}.$$

Another approach is to simply observe from the graph that the line goes down three units for each four units to the right.