MATH 251 (PHILLIPS) MIDTERM 0 VERSION 2, 9:00 am F 17 January 2025

NAME: SOLUTIONS Student id: $\pi\pi\pi$-$\pi\pi$-$\pi\pi\pi\pi$

INSTRUCTIONS: No books, notes, or calculators are permitted on this test. All answers must be simplified as much as possible. Write all answers in the spaces provided at the right. Do scratchwork on the back or on blank paper provided for this purpose. *No partial credit.* Time: 20 minutes.

1. Simplify the following expression as much as possible. If no simplification is possible, write "not possible": $\dfrac{e^{2y}+3}{e^{2y}+6}$

Solution: The expression $\dfrac{e^{2y}+3}{e^{2y}+6}$ can't be simplified.

2. Multiply out: $(q^2+2q-4)(q-2)$.

Solution:

$$(q^2+2q-4)(q-2) = q^3+2q^2-4q-2q^2-4q+8 = q^3-8q+8.$$

3. Let $f(x) = 3-x$. Evaluate the expression $f(2-x)-f(7x)$, and simplify it as much as possible.

Solution:

$$f(2-x)-f(x) = 3-(2-x)-(3-7x) = 3-2+x-3+7x = 8x-2.$$

4. Find all real solutions to the equation $\ln(2-3x)=3$. If no real solution exists, write "no solution".

Solution:

$$\begin{aligned} \ln(2-3x) &= 3 \\ 2-3x &= e^3 \\ 2-e^3 &= 3x \\ x &= \frac{2-e^3}{3}. \end{aligned}$$

The answer can be rewritten as $x = \frac{2}{3}-\frac{1}{3}e^3$, but this is not necessary. However, it can't be further simplified.

5. Find all real solutions to the equation $\dfrac{7x}{x^2+10} = -1$. If no real solution exists, write "no solution".

Solution:

$$\begin{aligned} \frac{7x}{x^2+10} &= -1 \\ 7x &= -x^2-10 \\ x^2+7x+10 &= 0 \\ (x+2)(x+5) &= 0 \\ x=-2 \qquad \text{or} &\qquad x=-5. \end{aligned}$$

Note that both answers actually are solutions to the original equation, that is, that multiplying both sides by x^2+10 at the first step did not introduce any extraneous solutions.

Since there is no partial credit, no credit is given for only one of the two solutions.

6. Write as a single fraction, and simplify as much as possible: $\dfrac{3}{y+6} - \dfrac{1}{y+3}$

Solution:

$$\frac{3}{y+6} - \frac{1}{y+3} = \frac{3(y+3)-(y+6)}{(y+6)(y+3)} = \frac{3y+6-y-6}{(y+6)(y+3)} = \frac{y+3}{(y+6)(y+3)}.$$

7. Simplify completely (for $y > 0$): $\dfrac{\left(\frac{2}{9\sqrt{y}}\right)}{\left(\frac{y^{3/2}}{6}\right)}$

Solution:

$$\frac{\left(\frac{2}{9\sqrt{y}}\right)}{\left(\frac{y^{3/2}}{6}\right)} = \left(\frac{2}{9y^{1/2}}\right)\left(\frac{6}{y^{3/2}}\right) = \frac{2\cdot 6}{9y^2} = \frac{4}{3y^2}.$$

If you want, you can also write the answer as $\frac{4}{3}y^{-2}$, but this is not necessary.

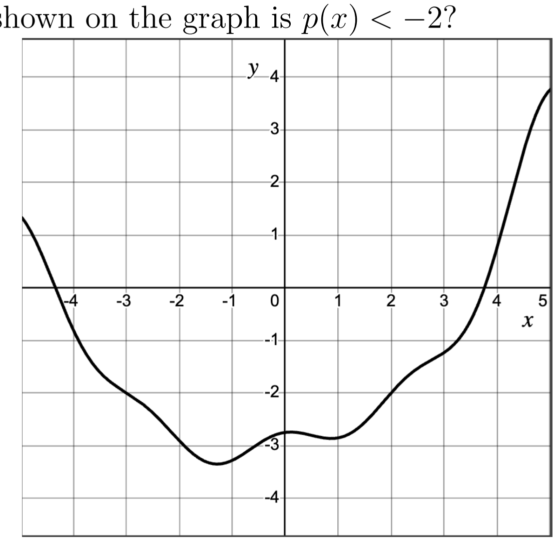
8. Find all real solutions to the equation $5\left(\frac{1}{x^2} - 3\right) = -15$. If no real solution exists, write "no solution".

Solution: Expand the left hand side, getting $\frac{5}{x^2} - 15 = -15$, that is, $\frac{5}{x^2} = 0$. Multiply both sides by x^2 to get $5 = 0$. Therefore there are no solutions. (Alternatively, clearly $5/x^2$ can never be zero.)

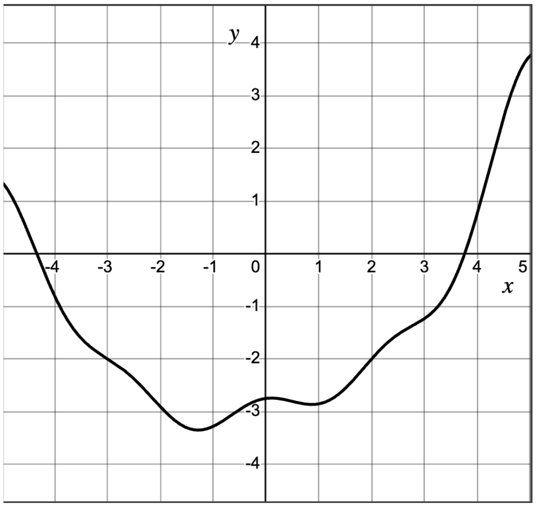
9. Find all real numbers c such that $(-c, 17)$ is in the first quadrant (and not on any of the coordinate axes).

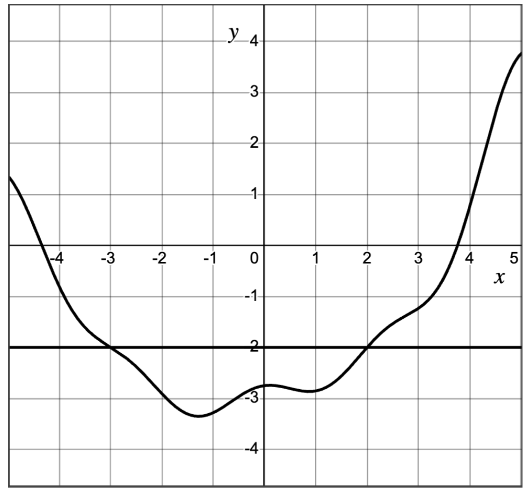
Solution: $(-c, 17)$ is in the first quadrant if and only if $-c > 0$, which happens if and only if $c < 0$.

10. The graph of a function $y = p(x)$ is sketched below (at the left). For which values of x shown on the graph is $p(x) < -2$?



Solution:





By reading the graph, we see that the function is on or above the line $y = -2$ for $-3 < x < 2$. (See the graph at the right above, on which this line is also shown.) So the answer is $-3 < x < 2$.

The answer $2 < x < -3$ is not correct, since there are no values of x which satisfy this inequality. Similarly, x in $(-3, 2)$ is a correct answer, but $(2, -3)$ is the empty set and is therefore not a correct answer.