MATH 251 (PHILLIPS) MIDTERM 0 EXTRA PROBLEM LIST SET 1 SOLUTIONS

Warning: Not enough proofreading has been done! (People have gotten extra credit for catching previous errors.)

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1. Simplify the following expression as much as possible. If no simplification is possible, write "not possible": $\dfrac{\sin(7x)+7}{\sin(7x)-7}$

Solution: The expression $\dfrac{\sin(7x)+7}{\sin(7x)-7}$ can't be simplified.

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2. Simplify completely (for $y>0$): $\dfrac{\left(\frac{2}{\sqrt{y}}\right)}{\left(\frac{y^{1/2}}{2}\right)}$

Solution:

$$\frac{\left(\frac{2}{\sqrt{y}}\right)}{\left(\frac{y^{1/2}}{2}\right)} = \left(\frac{2}{\sqrt{y}}\right)\left(\frac{2}{y^{1/2}}\right) = \frac{4}{\sqrt{y}\cdot y^{1/2}} = \frac{4}{y^{1/2}\cdot y^{1/2}} = \frac{4}{y}.$$

If you want, you can rewrite the answer as $4y^{-1}$, but that is not necessary.

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3. Find all real solutions to the equation $\dfrac{e^{-5x}}{x^2}=0$. If no real solution exists, write "no solution".

Solution: Multiply both sides by x^2 to get $e^{-5x}=0$. Since e^{-5x} can never be zerothere are no solutions.
(Alternatively, since e^{-5x} can never be zero, it is obvious that $\frac{e^{-5x}}{x^2}$ can never be zero.)

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4. Let $g(x)=7-4x$. Evaluate the expression $\dfrac{g(5+h)-g(5)}{h}$, and simplify it as much as possible.

Solution:
$$\frac{g(5+h)-g(5)}{h} = \frac{7-4(5+h)-(7-4\cdot 5)}{h} = \frac{7-20-4h-7+20}{h} = \frac{-4h}{h} = -4.$$

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5. Write as a single fraction, and simplify as much as possible: $\dfrac{3}{x+4}-\dfrac{1}{x-5}$

Solution:
$$\frac{3}{x+4}-\frac{1}{x-5} = \frac{3(x-5)-(x+4)}{(x+4)(x-5)} = \frac{3x-15-x-4}{(x+4)(x-5)} = \frac{x-19}{(x+4)(x-5)}.$$

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6. Find all real solutions to the equation $\dfrac{8}{x}-x=-2$. If no real solution exists, write "no solution".

Solution:
$$\begin{aligned}\frac{8}{x}-x&=-2\\ 8-x^2&=-2x\\ x^2-8&=2x\end{aligned}$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = 4 \qquad \text{or} \qquad x = -2.$$

Note that both answers actually are solutions to the original equation, that is, that multiplying both sides by x at the first step did not introduce any extraneous solutions.

Since there is no partial credit, no credit is given for only one of the two solutions.

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7. Find all real solutions to the equation $\ln(7-2x)+2=0$. If no real solution exists, write "no solution".

Solution:

$$\ln(7-2x)+2=0$$

$$\ln(7-2x) = -2$$

$$7-2x = e^{-2}$$

$$7-e^{-2} = 2x$$

$$x = \frac{7-e^{-2}}{2}$$

If you want, you can rewrite the anwer as $x = \frac{7}{2} - \frac{1}{2}e^{-2}$, but that is not necessary. However, it can't be further simplified.

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8. Find the domain of the function $r(x) = (-x)^{-1/4}$.

Solution: Rewrite $r(x) = \dfrac{1}{\sqrt[4]{-x}}$. We can now see that x is in the domain if and only if $-x \geq 0$ (so that $\sqrt[4]{-x}$ is defined) and $\sqrt[4]{-x} \neq 0$ (so that $\dfrac{1}{\sqrt[4]{-x}}$ is also defined). The first condition requires $x \leq 0$, and the second rules out $x = 0$, so the answer is all real x with $x < 0$.

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9. Multiply out: $\quad (w-1)\big(w^2 - 4w - 3\big)$.

Solution:

$$(w-1)\big(w^2-4w-3\big) = w^3 - w^2 - 4w^2 + 4w - 3w + 3 = w^3 - 5w^2 + w + 3.$$

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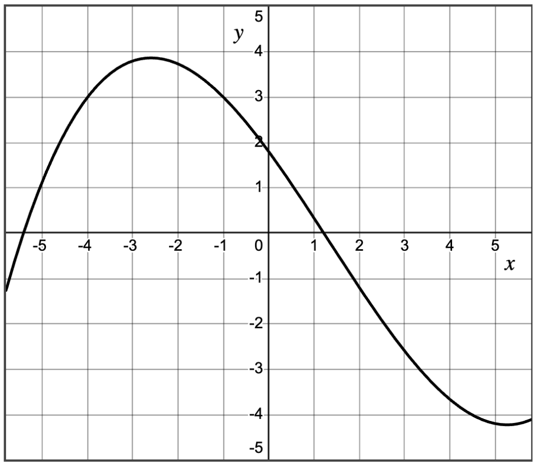
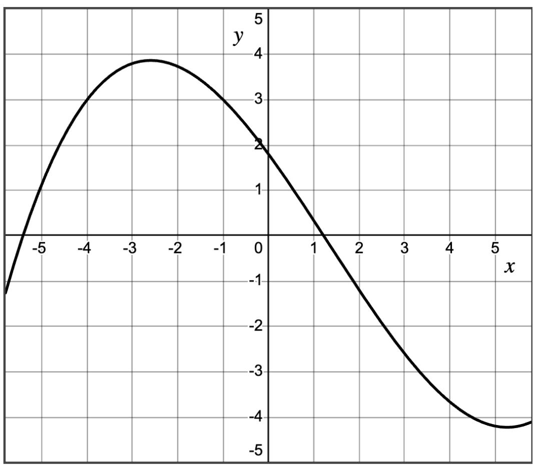
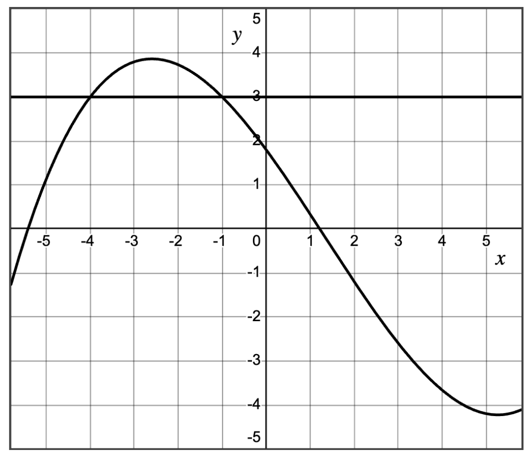
10. The graph of a function $y = g(x)$ is sketched below (at the left). For which values of x shown on the graph is $g(x) > 3$?



Solution:





By reading the graph, we see that the function is on or above the line $y = 3$ for $-4 < x < -1$. (See the graph at the right above, on which this line is also shown.) So the answer is $-4 < x < -1$.

The answer $-1 < x < -4$ is not correct, since there are no values of x which satisfy this inequality. Similarly, x in $(-4, -1)$ is a correct answer, but $(-1, -4)$ is the empty set and is therefore not a correct answer.