MATH 251 (PHILLIPS) MIDTERM 0 EXTRA PROBLEM LIST SET 2 SOLUTIONS

Warning: Not enough proofreading has been done! (People have gotten extra credit for catching previous errors.)

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1. Find all real solutions to the equation $\dfrac{6}{x} + \dfrac{7}{x^2} = 1$. If no real solution exists, write "no solution".

Solution: This is a quadratic equation in $1/x$:

$$\frac{6}{x} + \frac{7}{x^2} = 1$$
$$7\left(\frac{1}{x}\right)^2 + 6\left(\frac{1}{x}\right) - 1 = 0$$
$$\left(7\left(\frac{1}{x}\right) - 1\right)\left(\frac{1}{x} + 1\right) = 0$$
$$\frac{1}{x} = \frac{1}{7} \quad \text{or} \quad \frac{1}{x} = -1.$$
$$x = -1 \quad \text{or} \quad x = 7.$$

Note that both answers actually are solutions to the original equation, that is, that multiplying both sides by x at the last step did not introduce any extraneous solutions.

Since there is no partial credit, no credit is given for only one of the two solutions.

Alternate solution: Multiply through by x^2 first, getting:

$$6x + 7 = x^2$$
$$x^2 - 6x - 7 = 0$$
$$(x+1)(x-7) = 0$$
$$x = -1 \quad \text{or} \quad x = 7.$$

Note that both answers actually are solutions to the original equation, that is, that multiplying both sides by x^2 at the first step did not introduce any extraneous solutions.

Since there is no partial credit, no credit is given for only one of the two solutions.

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2. Simplify the following expression as much as possible. If no simplification is possible, write "not possible": $\dfrac{x^3 + 7x}{x^3 + 2x}$

Solution:

$$\frac{x^3 + 7x}{x^3 + 2x} = \frac{x(x^2 + 7)}{x(x^2 + 2)} = \frac{x^2 + 7}{x^2 + 2}.$$

The last expression can't be further simplified.

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3. Simplify completely (for $x > 0$): $\dfrac{(2\sqrt{x})^3}{(2x^{3/2})^2}$

Solution:

$$\frac{(2\sqrt{x})^3}{(2x^{3/2})^2} = \frac{(2x^{1/2})^3}{(2x^{3/2})^2} = \frac{2^3 x^{3/2}}{2^2 x^3} = \frac{2^{3-2}}{x^{3-3/2}} = \frac{2}{x^{3/2}}.$$

If you want, you can rewrite the answer as $2x^{-3/2}$, but that is not necessary.

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4. Multiply out: $(y-5)\big(y^2 + 3y - 2\big)$.

Solution:

$$(y-5)\big(y^2 + 3y - 2\big) = y^3 - 5y^2 + 3y^2 - 15y - 2y + 10 = y^3 - 2y^2 - 17y + 10.$$

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5. Let $f(x) = 7 - x$. Evaluate the expression $f(2-x) - f(x)$, and simplify it as much as possible.

Solution:

$$f(2-x) - f(x) = 7 - (2-x) - (7-x) = 7 - 2 + x - 7 + x = 2x - 2.$$

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6. Find all real numbers a such that $|a+2| = -a-2$.

Solution: $|a+2| = -a-2$ if and only if $|a+2| = -(a+2)$. Since $|x| = -x$ if and only if $x \leq 0$, this happens if and only if $a+2 \leq 0$, which is true if and only if $a \leq -2$.

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7. Find all real solutions to the equation $3y^{-3} = 0$. If no real solution exists, write "no solution".

Solution: Multiply both sides by y^3 to get $3 = 0$. Therefore there are no solutions. (Alternatively, write $3y^{-3} = 3/y^3$, which can obviously never be zero.)

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8. Find all real solutions to the equation $4e^{-3x} + 11 = 3$. If no real solution exists, write "no solution".

Solution:

$$\begin{aligned} 4e^{-3x} + 11 &= 3 \\ 4e^{-3x} &= -8 \\ e^{-3x} &= -8 \end{aligned}$$

Since $e^{-3x} = 1/e^{3x}$ is never negative, there are no real solutions.

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9. Write as a single fraction, and simplify as much as possible: $\dfrac{2}{p+4} - \dfrac{1}{p+5}$

Solution:

$$\frac{2}{p+4} - \frac{1}{p+5} = \frac{2(p+5)-(p+4)}{(p+4)(p+5)} = \frac{2p+10-p-4}{(p+4)(p+5)} = \frac{p+6}{(p+4)(p+5)}.$$

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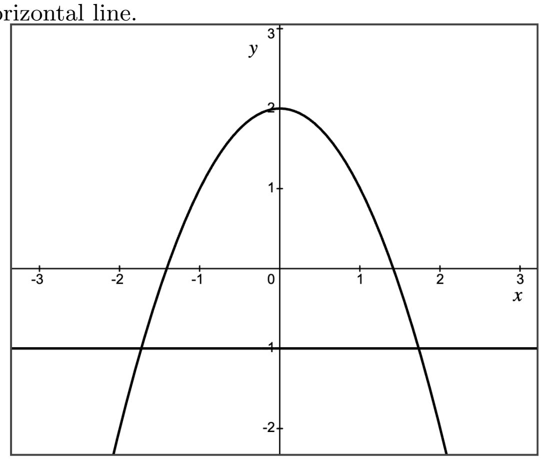
10. The curve in the graph below (at the left) is the graph of the function $y = 2x^2$. Find the **exact** values of **both** coordinates of **all** points at which this curve intersects the horizontal line.



Solution: There are clearly two points. The equation of the line is $y = -1$. Therefore the y-coordinates are both -1, and the x-coordinates are $\sqrt{3}$ and $-\sqrt{3}$ (the solutions to $2?x^2 = -1$). The points are therefore $\left(\sqrt{3}, -1\right)$ and $\left(-\sqrt{3}, -1\right)$.

Since there is no partial credit, no credit will be given if one of the points is omitted, or if only one coordinate of each point is given.