Math 251 Midterm 1 NAME:_____
W 12 Feb. 2025, 11:00 am. Student ID:_____
GENERAL INSTRUCTIONS

1. DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
2. The exam pages are **two sided**.
3. Closed book, except for a 3×5 file card, written on both sides.
4. The following are all prohibited: Calculators (of any kind), cell phones, laptops, iPods, electronic dictionaries, and any other electronic devices or communication devices. All electronic or communication devices you have with you must be turned completely off and put inside something (pack, purse, etc.) and out of sight.
5. The point values are as indicated in each problem; total 100 points.
6. Write all answers on the test paper. Use the bottom of page 5 for long answers or scratch work. (If you do write an answer there, indicate on the page containing the problem where your answer is.)
7. Show your work. You must state what you did, legibly, clearly, correctly, and using correct notation. Among many other things, this means putting "=", limit symbols, etc. in all places where they belong, and not in any places where they don't belong. It also means organizing your work so that the order of the steps is clear, and it is clear how the steps are related to each other.
8. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Cross out any work you do not want considered. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, or for which the work is riddled with notation errors, will receive little or no credit.
9. Be sure you say what you mean. Credit will be based on what you say, not what you mean.
10. When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(23)$, or $\frac{2\pi}{9}$. Decimal approximations will not be accepted.
11. Final answers must always be simplified unless otherwise specified.
12. Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).
13. Time: 50 minutes.

1	2	3	4	5	6	7	8	9	10	TOTAL
1	6	10	10	10	10	20	13	10	10	100

1. (1 point.) True or false: Inverse functions were invented by the Devil.

2. (6 points) State carefully the definition of the derivative of a function.

3. (a) (9 points) If $f(x) = 5x - x^2$, compute the derivative $f'(4)$ *directly from the definition of the derivative* (which you are supposed to have given above). (No credit will be given for just using the differentiation rules, but see Part (b.)

(b) (1 point) Use the differentiation rules we have learned to check your answer to part (a).

4. (10 points) Differentiate te function $L(x) = \dfrac{\sin(x)}{e^x + x^4 + 15} + \ln(2)$. (You need not do this directly from the definition.)

5. (10 points) Differentiate the function $h(x) = (x^5 + 3x)^8$. (You need not do this directly from the definition.)

6. (10 points) Find the exact value of $\lim_{x \to -2} \dfrac{x^2 - 4x - 12}{7(x+2)}$, or explain why this limit does not exist.

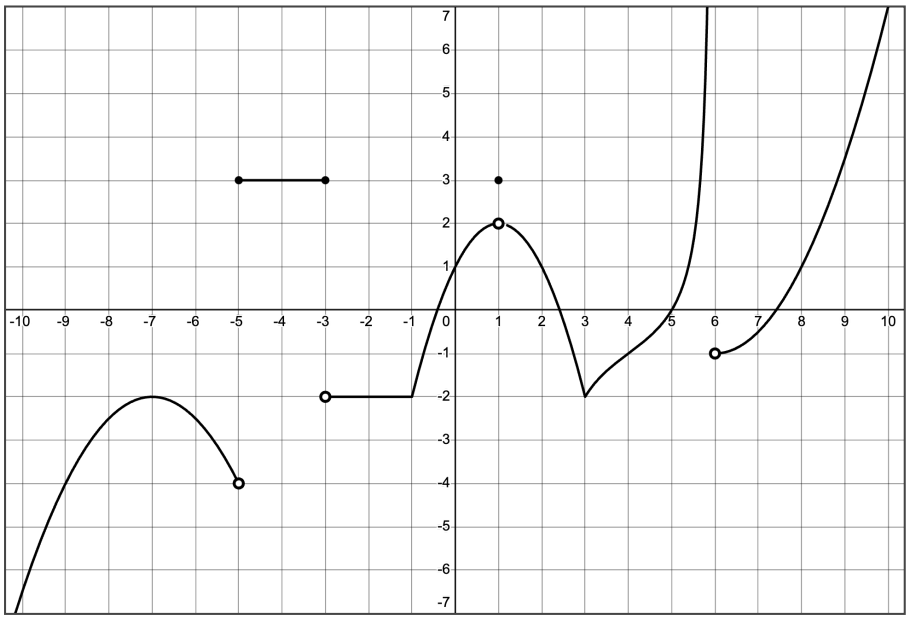
7. (20 points.) A rectangular box is to have a base that is twice as long as it is wide. 12 square feet of material are available to make the box (including the top). Find the dimensions of the box which maximizes the volume.

Include units. Be sure to verify that your maximum or minimum really is what you claim it is, using methods we have see so far (not the second derivative test).

8. (13 points) Use the methods of calculus to find the exact values of x at which the function $q(x) = x^3 - 12x + 20$ takes its absolute minimum and maximum on the interval $[-3, 0]$.

(No credit will be given for correct guesses without supporting work that is valid for general functions of the sort considered in this course.)

9. (5 points/part) For the function $y = L(x)$ graphed below, answer the following questions:



(a) Does $\lim_{x \to 1} L(x)$ exist? If so, what is it? If not, why not?

(b) Is L continuous at -1? Why or why not?

(Continued on back or on next page.)

10. (10 points.) This problem is about using correct notation. Accordingly, almost all the credit is for correctness of notation.

Consider the problem of finding the exact value of $\lim_{x\to -1} \dfrac{x^3-2x^2-3x}{x+1}$. The method is to factor the numerator and cancel one of the factors. The factors of the numerator are $x+1$, x, and $x-3$. (You need not check these.)

Write out the calculation in full, in correct notation which exhibits correctly the steps of the calculation. In particular, put "=" and "lim" everywhere they belong, and nowhere else. Start by writing $\lim_{x\to -1} \dfrac{x^3-2x^2-3x}{x+1}$. Show at least the following steps (not labelled):

- After factoring but before cancellation.
- After cancellation but before substituting $x=-1$.
- After substituting $x=-1$ but before possible simplification.
- The simplified final result, if the result in the previous step can be simplified.