MIDTERM 1 REVIEW SESSION WORKSHEET FOR W 2025

It actually has 103 points.

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1. (5 points) State carefully the definition of the derivative of a function.

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2. (a) (9 points) If $f(x) = 7 - x^2$, compute the derivative $f'(2)$ *directly from the definition*. (No credit will be given for just using the differentiation rules, but see Part (b).)

(b) (1 point) Use the differentiation rules we have learned to check your answer to part (a).

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3. (9 points.) Differentiate the function $w(t) = t^2 \cos(t) - \dfrac{5}{\sqrt{t}} + \dfrac{7}{71}$.

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4. (9 points.) Differentiate the function $h(s) = \sqrt{12 + s^2 + \sin(s)}$.

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5. (20 points) The Wang Container Corporation plans to manufacture wooden boxes with square bases and hinged lids. The wood for the bottom and sides costs \$3 per square foot, and the wood for the lid costs \$1 per square foot. [Evidently the lid will be rather flimsy.] Furthermore, each box requires hinges and a latch costing a total of \$6. If the total cost of the materials is only allowed to be \$54, what are the dimensions of the largest volume box that can be manufactured?

Include units, and be sure to verify that your maximum or minimum really is what you claim it is.

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6. (15 points) Use the methods of calculus to find the exact values of x at which the function $f(x) = \dfrac{x}{x^2 + 1}$ takes its absolute minimum and maximum on the interval $[0, 7]$.

(No credit will be given for correct guesses without supporting work that is valid for general functions of the sort considered in this course.)

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7. (9 points.) Find the exact value of $\displaystyle\lim_{x \to -2} \frac{x + 2}{x^2 - 3x - 10}$, or explain why it does not exist:

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8. (9 points.) Find the exact value of $\displaystyle\lim_{x \to -1} \frac{x^2 - x - 19}{x - 2}$, or explain why it does not exist:

9. (9 points.) This problem is about using correct notation. Accordingly, almost all the credit is for correctness of notation.

Consider the problem of finding the exact value of $\lim\limits_{x \to 2} \dfrac{x^4 - 2x^3 - x\sin(x) + 2\sin(x)}{x - 2}$. The method is to factor the numerator (the factors are $x - 2$ and $x^3 - \sin(x)$) and cancel the factor $x - 2$, getting a limit which can be found by direct substitution.

Write out the calculation in full, in correct notation which exhibits correctly the steps of the calculation. In particular, put "=" and "lim" everywhere they belong, and nowhere else. Start by writing $\lim\limits_{x \to 2} \dfrac{x^4 - 2x^3 - x\sin(x) + 2\sin(x)}{x - 2}$. Show at least the following steps:

(1) After factoring but before cancellation.
(2) After cancellation but before substituting $x = 2$.
(3) After substituting $x = 2$ but before possible simplification.
(4) The simplified final result, if the result in the previous step can be simplified. (Don't give a decimal approximation to $\sin(2)$.)

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10. (4 points/part) For the function $y = k(x)$ graphed below, answer the following questions:
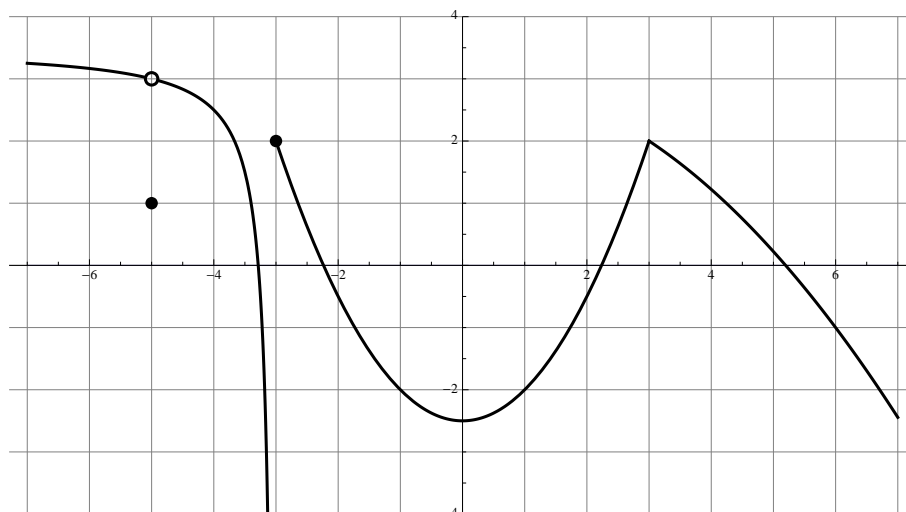
(a) List all numbers a in $(-7, 7)$ such that k is not differentiable at a. Give reasons.

(b) List all numbers a in $(-7, 7)$ such that $\lim_{x \to a} k(x)$ does not exist.