Math 251 Midterm 1 NAME:________________________

F 31 Jan. 2025, 9:00 am. Student ID:________________________

GENERAL INSTRUCTIONS

1. DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
2. The exam pages are **two sided**.
3. Closed book, except for a 3×5 file card, written on both sides.
4. The following are all prohibited: Calculators (of any kind), cell phones, laptops, iPods, electronic dictionaries, and any other electronic devices or communication devices. All electronic or communication devices you have with you must be turned completely off and put inside something (pack, purse, etc.) and out of sight.
5. The point values are as indicated in each problem; total 100 points.
6. Write all answers on the test paper. Use the bottom of page 5 for long answers or scratch work. (If you do write an answer there, indicate on the page containing the problem where your answer is.)
7. Show your work. You must state what you did, legibly, clearly, correctly, and using correct notation. Among many other things, this means putting "=", limit symbols, etc. in all places where they belong, and not in any places where they don't belong. It also means organizing your work so that the order of the steps is clear, and it is clear how the steps are related to each other.
8. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, or for which the work is riddled with notation errors, will receive little or no credit.
9. Be sure you say what you mean. Credit will be based on what you say, not what you mean.
10. When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(23)$, or $\frac{2\pi}{9}$. Decimal approximations will not be accepted.
11. Final answers must always be simplified unless otherwise specified.
12. Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).
13. Time: 50 minutes.

1	2	3	4	5	6	7	8	9	10	TOTAL	EC
1	6	10	10	10	10	20	13	10	10	100	

1. (1 point.) The Squeeze Theorem was invented for the purpose of torturing which of:

    1. Functions.

    2. Calculus students.

    3. Professors who have to teach it in calculus classes.

2. (6 points) State carefully the definition of the derivative of a function.

3. (a) (9 points) If $f(x) = x^2 + 3$, compute the derivative $f'(4)$ *directly from the definition of the derivative* (which you are supposed to have given above). (No credit will be given for just using the differentiation rules, but see Part (b.)

   (b) (1 point) Use the differentiation rules we have learned to check your answer to part (a).

4. (10 points) Differentiate the function $q(x) = (3x^2 - 16x)\sin(x) + \frac{1}{11}$. (You need not do this directly from the definition.)

5. (10 points) Differentiate the function $h(x) = \cos(4x^3 + 8x)$. (You need not do this directly from the definition.)

6. (10 points) Find the exact value of the limit $\lim_{x \to 6} \frac{x^2 - 36}{x^2 - 5x - 6}$, or explain why this limit does not exist.

7. (20 points.) A farmer wants to build a rectangular fenced enclosure. Because of bizarre local laws, the east and west fences will cost 6 florins per meter, the north fence will cost 3 florins per meter, and the south fence will cost 1 florin per meter. The farmer has 2400 florins available to build the enclosure. Use calculus to find the lengths of the south and west fences of the largest such enclosure that can be built. Include units. Verify that the solution corresponds to the largest possible area, using methods we have seen so far in the course (not the second derivative test).

8. (13 points) Use the methods of calculus to find the exact values of x at which the function $f(x) = x^3 - 3x + 7$ takes its absolute minimum and maximum on the interval $[0, 3]$.

(No credit will be given for correct guesses without supporting work that is valid for general functions of the sort considered in this course.)

9. (10 points.) This problem is about using correct notation. Accordingly, almost all the credit is for correctness of notation.

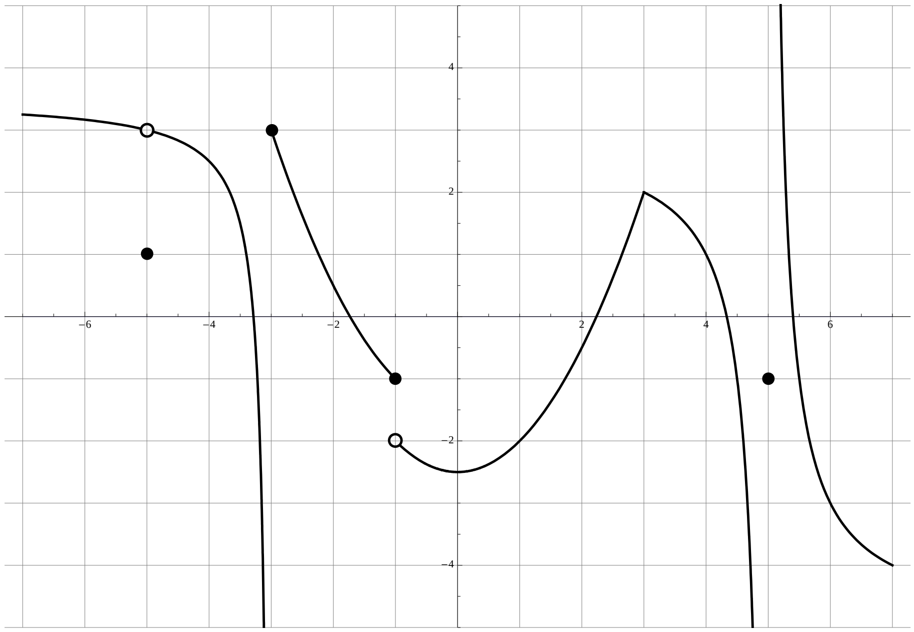
Consider the problem of finding the exact value of $\lim_{x \to -4} \frac{x^3 + 4x^2 + 9x + 36}{x + 4}$. The method is to factor the numerator and cancel one of the factors. The factors of the numerator are $x + 4$ and $x^2 + 9$. (You need not check this factorization.)

Write out the calculation in full, in correct notation which exhibits correctly the steps of the calculation. In particular, put "=" and "lim" everywhere they belong, and nowhere else. Start by writing $\lim_{x \to -4} \frac{x^3 + 4x^2 + 9x + 36}{x + 4}$. Show at least the following steps:

- After factoring but before cancellation.
- After cancellation but before substituting $x = -4$.
- After substituting $x = -4$ but before possible simplification.
- The simplified final result, if the result in the previous step can be simplified.

(Continued on back or on next page.)

10. (5 points/part) For the function $y = g(x)$ graphed below, answer the following questions:



(a) Does $\lim_{x\to -1} g(x)$ exist? If so, what is it? If not, why not?

(b) Which of the following best describes $g'(1)$? Why?

(1) $g'(1)$ does not exist.
(2) $g'(1)$ is close to 0.
(3) $g'(1)$ is positive and not close to 0.
(4) $g'(1)$ is negative and not close to 0.
(5) None of the above.

Extra credit. (15 extra credit points; grading will be harsher than on related problems on the main exam. Do not attempt this problem until you have done and checked your answer to all the ordinary problems on this exam. It will only be counted if you get a grade of B or better on the main part of this exam.)

Let f be a function such that $f'(t) = \sqrt[4]{27 + \arctan(6t)}$. Let

$$g(x) = \cos\left(\left[\left([f(x)+7]^{26} + 11\right)^{1/13} + e^{2x}\right]^{99}\right).$$

Find $g'(x)$.