Math 251 Final Exam NAME:_____
Thursday 7 December 2017 Student ID:_____

GENERAL INSTRUCTIONS

1. DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
2. The exam pages are **two sided**.
3. Closed book, except for a 3×5 file card, written on both sides.
4. The following are all prohibited: Calculators (of any kind), cell phones, laptops, iPods, electronic dictionaries, and any other electronic devices or communication devices. All electronic or communication devices you have with you must be turned completely off and put inside something (pack, purse, etc.) and out of sight.
5. The point values are as indicated in each problem; total 200 points.
6. Write all answers on the test paper. If you use scratch paper, be sure to write "see scratch" in the space for the answer to the problem, and write your name and the problem number on every piece of scratch paper you want to have graded.
7. Show your work. You must state what you did, legibly, clearly, correctly, and using correct notation. Among many other things, this means putting "=", limit symbols, etc. in all places where they belong, and not in any places where they don't belong. It also means organizing your work so that the order of the steps is clear, and it is clear how the steps are related to each other.
8. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, or for which the work is riddled with notation errors, will receive little or no credit.
9. Be sure you say what you mean. Credit will be based on what you say, not what you mean.
10. When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(23)$, or $\frac{2\pi}{9}$. Decimal approximations will not be accepted.
11. Final answers must always be simplified unless otherwise specified.
12. Final exams will be available for inspection after they are graded, but I keep the originals. (You may get a copy on request.) Grading complaints must be made before final grades are turned in.
13. Time: 120 minutes (unless extended, with class agreement, by an early start or late finish).

DO NOT WRITE BELOW THIS LINE

1	2	3	4	5	6	7	8	9	TOTAL	EC
40	10	12	30	30	11	15	17	35	200	

1. (10 points/part) Find the exact values of the following limits (possibly including ∞ or $-\infty$), or explain why they do not exist or there is not enough information to evaluate them. Give justification in all cases (not just heuristic arguments). Remember to use correct notation.

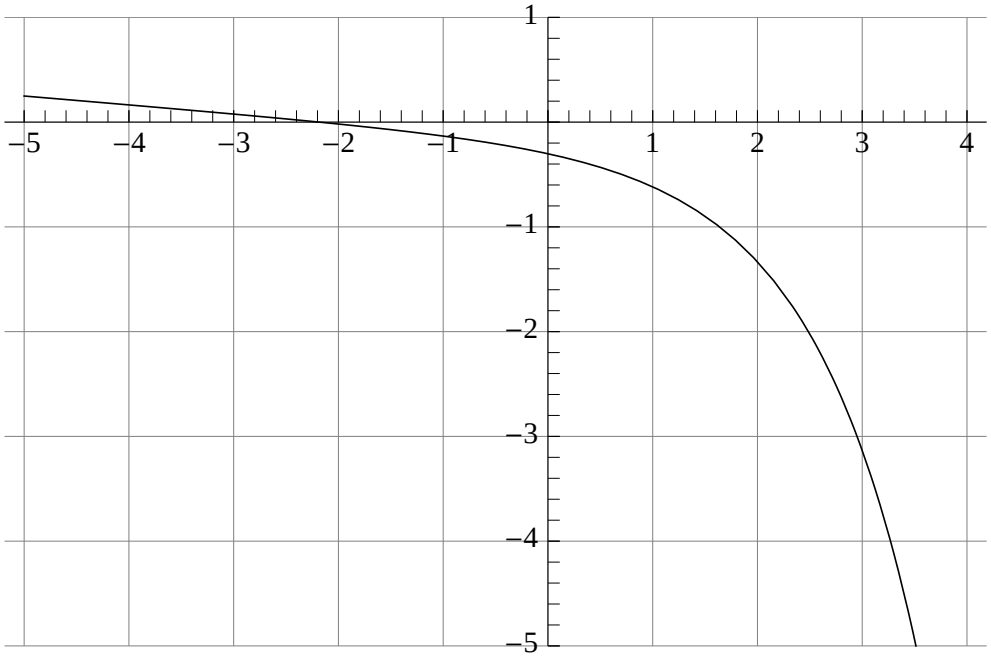
(a) $\lim_{x \to 3} \dfrac{x^2 - 9}{\cos(x) - 3}$.

(b) $\lim_{y \to \infty} \dfrac{2y + 216}{13y - 5\sin(y)}$. (Be sure to show your work!)

(c) $\lim_{x \to 0} \dfrac{5x^2}{1 - \cos(7x)}$.

(d) $\lim_{x \to 2^-} \dfrac{e^{-x}}{x - 2}$. (Be sure to show your work!)

2. (10 points.) The picture below shows the graph of a function f. Suppose that Newton's method is used to approximate the root of the equation $f(x) = 0$ which is visible in the picture, with initial approximation $x_1 = 3$. Draw on the graph the tangent lines that are used to find x_2 and x_3, and estimate the numerical values of x_2 and x_3.



3. (12 points) Find the equation of tangent line to the graph of $g(x) = 2x + 4\sqrt{3x - 2}$ at $x = 2$. You need not calculate the derivative directly from the definition.

4. (10 points/part) Differentiate the functions as requested.

(a) Find $f'(x)$, where $f(x) = \pi^3 + \dfrac{2x+1}{x^2+1}$.

(b) Let $f(t) = e^{7t+\arcsin(t)} + \csc\left(\frac{\pi}{3}\right)$. Find $f'(t)$.

(c) Let $q(x) = \ln(x)\cos(x^2 - cx)$, where c is a constant. Find $q'(x)$.

5. (30 points.) A 5 meter ladder leans against a vertical wall in a room with a high ceiling and level floor. Because the floor is slippery, the foot of the ladder is sliding away from the wall. When the foot is 3 meters from the wall, it is sliding away at 7 meters per hour. At this time, how fast is the top of the ladder sliding down the wall? (Be sure to include correct units.)

6. A fire-breathing monster is thrown upwards on the planet Yuggxth. Its height t seconds after it is thrown is $40t - 5t^2$ feet, until it hits the ground again.

(a) (4 points) Is the monster falling or rising 6 seconds after being thrown? How fast?

(b) (7 points) How long after being thrown does the monster reach its maximum height?

7. (14 points) If $y^3 = \sin(11x - y) - \sin(7)$, find $\dfrac{dy}{dx}$ by implicit differentiation. (You must solve for $\dfrac{dy}{dx}$.)

8. (17 points) Suppose we know the following about the function h:

h is defined and continuous on $(-\infty, \infty)$, and $h'(x)$ and $h''(x)$ exist on all of $(-\infty, \infty)$.

h has only one critical number, namely 1.

$h(1) \approx -2.718$.

$h'(x) < 0$ for x in the interval $(-\infty, 1)$.

$h'(x) > 0$ for x in the interval $(1, \infty)$.

The only solution to $h''(x) = 0$ is $x = 0$.

$h(0) = -2$.

$h''(x) < 0$ for x in the interval $(-\infty, 0)$.

$h''(x) > 0$ for x in the interval $(0, \infty)$.

$\lim_{x \to -\infty} h(x) = 0$.

Find the asymptotes, intervals of increase and decrease, local minimums and maximums, intervals of concavity up and down, and inflection points. Then draw the graph of h. Make sure that the graph matches the information about concavity etc. that you found.

9. (35 points.) You have a magical animal which is much stronger when going north than in any other direction. You want to construct a rectangular enclosure for this animal. The fence on the north side costs 6 florins per yard, and the fence on the south, west, and east sides costs 2 florins per yard. The area of the enclosure is to be 200 square yards. What are the dimensions of the cheapest possible enclosure?

Include units, and be sure to verify that your maximum or minimum really is what you claim it is.

(Extra credit on next page.)

Extra credit. (Do not attempt these problems until you have done and checked your answer to all the ordinary problems on this exam. They will only be counted if you get 150 points or more on the main part of this exam, and also only if your course grade is B- or better without extra credit.)

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EC1. (15 extra credit points; grading will be harsher than on related problems on the main exam.) Let f be a function such that $f'(t) = \sqrt[3]{7 + \arctan(t)}$. Let

$$g(x) = \sin\left(\left[\left([f(x) + 7]^{16} + 11\right)^{1/12} + e^{2x}\right]^{99}\right).$$

Find $g'(x)$.

EC2. (15 extra credit points) Find a real number r such that

$$\lim_{x \to 0} \frac{\sin(x) - (x + rx^3)}{x^5}$$

exists (in particular, is not infinite), and for this choice of r find the limit above.

EC3. (25 extra credit points) A four dimensional box has a cubical base and no top. Its four dimensional volume is supposed to be $8\,\text{ft}^4$. What dimensions minimize the volume of material needed to make its base and sides?

Hint: A box in four dimensional space has 8 three dimensional "faces", each of which has the shape of a three dimensional box.