Math 251 Midterm 2 NAME:____________________

W 16 November 2017 Student ID:____________________

GENERAL INSTRUCTIONS

1. DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
2. The exam pages are **two sided**.
3. Closed book, except for a 3×5 file card, written on both sides.
4. The following are all prohibited: Calculators (of any kind), cell phones, laptops, iPods, electronic dictionaries, and any other electronic devices or communication devices. All electronic or communication devices you have with you must be turned completely off and put inside something (pack, purse, etc.) and out of sight.
5. The point values are as indicated in each problem; total 100 points.
6. Write all answers on the test paper. Use the back of the last page for long answers or scratch work. (If you do write an answer there, indicate on the page containing the problem where your answer is.)
7. Show your work. You must state what you did, legibly, clearly, correctly, and using correct notation. Among many other things, this means putting "=", limit symbols, etc. in all places where they belong, and not in any places where they don't belong. It also means organizing your work so that the order of the steps is clear, and it is clear how the steps are related to each other.
8. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, or for which the work is riddled with notation errors, will receive little or no credit.
9. Be sure you say what you mean. Credit will be based on what you say, not what you mean.
10. When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(23)$, or $\frac{2\pi}{9}$. Decimal approximations will not be accepted.
11. Final answers must always be simplified unless otherwise specified.
12. Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).
13. Time: 50 minutes.

1	2	3	4	5	TOTAL	EC
25	10	10	25	30	100	

1. (25 points) A large spherical snowball was melting. (A child took it inside the house at 11:00 am, and his parents had not yet noticed.) At 11:07 am, its radius was 30 cm, and was decreasing at $\frac{1}{3}$ cm per minute. Was its surface area increasing or decreasing? At what rate? (Be sure to include the correct units in your answer.)

2. (10 points) Let $b(z) = \arctan(11) - z^2 \cos(\ln(z))$. Find $b'(z)$.

3. The function $f(x)$ satisfies the following three properties:

$$f(3) = -2, \quad f'(3) = 0.50, \quad \text{and} \quad f''(3) = 1.02.$$

(a) (6 points) Use the linearization (tangent line approximation) to estimate the value of $f(2.96)$. (You might not need all the information given. You answer will likely use a decimal point, but is must be exact.)

(b) (4 points) Do you expect your answer in part (a) to be too big or too small? Why? (Drawing a picture might be useful.)

4. (25 points) James Tutt Snodgrass III has finished Math 251 and gone home for the holidays to his mother's farm, only to be asked to solve the following problem:

His mother wants to fence off a rectangular enclosure for a llama. The west side of the enclosure will be a wall which is already present, but new fence must be built for the other three sides. She can afford 40 meters of fence. What is the largest possible total area she can enclose?

Include units, and be sure to verify that your maximum or minimum really is what you claim it is.

(Show a full mathematical solution. A correct guess with no valid work will receive no credit, and a correct number supported only by heuristic reasoning will receive very little credit.)

5. (30 points) Graph the function $f(x) = x^3 + 3x^2$ using the methods of calculus. In particular, determine exactly the x-intercept(s), y-intercept(s), asymptotes, intervals of increase and decrease, critical numbers, local maximums, local minimums, intervals of concavity, and inflection points. Be sure to label your axes, and include the scales. **Be sure to organize your work so that it is easy to follow, and explain what you are doing. A list of equations, with no explanation of how they relate to each other or to the problem, will receive little credit.**

Fill in the table (writing "none" if appropriate), and show full work and the graph elsewhere on the page.

Domain.	
x-intercepts.	
y-intercepts.	
Horizontal asymptotes.	
Vertical asymptotes.	
Critical numbers.	
Local minimums.	
Local maximums.	
Intervals of strict increase.	
Intervals of strict decrease.	
Intervals of upwards concavity.	
Intervals of downwards concavity.	
Inflection points.	

(Extra credit problems on the back of this page.)

Extra credit. (Do not attempt these problems until you have done and checked your answer to all the ordinary problems on this exam. They will only be counted if you get 75 or more points on the main part of this exam.)

EC1. (20 extra credit points.) A 5 meter long ladder leans against a vertical wall in a room with a high ceiling and level floor. Because the floor is slippery, the foot of the ladder is sliding away from the wall. When the foot of the ladder is 3 meters from the wall, it is sliding away at $\frac{4}{3}$ meters per hour, and this motion is slowing down at $\frac{1}{3}$ meters/hour^2. At this time, is the motion of the top of the ladder speeding up or slowing down? How fast? (Be sure to include correct units.)

EC2. (20 extra credit points.) Either draw the graph of a function satisfying the given conditions, or prove that no such function exists: $f''(x)$ exists for all $x > 1$, f is concave up on $[1, \infty)$, $\lim_{x\to\infty} f(x) = 3$, and $f(x) < 3$ for all $x > 1$. (Vague explanations and graphs not satisfying all the conditions will get no credit.)