WORKSHEET: THE FALLING CALCULUS BOOK

Names and student IDs: _____

The gravitational constant at the surface of the planet Yuggxth is exactly $2\,\mathrm{m/sec^2}$. It has no atmosphere, so falling objects encounter no friction. A calculus book is dropped down a vertical hole at time $t = 0\,\mathrm{sec}$, so its height at time $t\,\mathrm{sec}$ is $p(t) = -t^2\,\mathrm{m}$ above the surface. (Nobody on Yuggxth knows any calculus.) Note: $-t^2$, because the calculus book is falling *down*.

We want to know the velocity of the calculus book at time $t = 2\,\mathrm{sec}$. (This is analogous to the reading of the speedometer in a car.)

As a first approximation, let's find the average velocity over the time interval $[2, 4]$ (in seconds). It is

$$\frac{p(4) - p(2)}{4 - 2} = \frac{-4^2 - (-2^2)}{4 - 2} = \frac{-12}{2} = -6\,\mathrm{m/sec}.$$

The answer is negative because the calculus book is moving down and we measured height in meters *above* the surface. Thus, the calculus book is is rising at $-6\,\mathrm{m/sec}$, or falling at $6\,\mathrm{m/sec}$. Keep the signs right!

Use the same method to find the average velocity over the time interval $[2, 3]$ (in seconds).

Use the same method to find the average velocity over the time interval $[2, 2.1]$ (in seconds). (Feel free to use a calculator on this one.)

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Use the same method to find the average velocity over the time interval $[2, 2.01]$ (in seconds). (Feel free to use a calculator on this one.)

Use the same method to find the average velocity over the time interval $[1, 2]$ (in seconds).

Use the same method to find the average velocity over the time interval $[1.9, 2]$ (in seconds). (Feel free to use a calculator on this one.)

What goes wrong if you try find the velocity at $t = 2\,\text{sec}$ by to taking both times to be $t = 2\,\text{sec}$?

Nevertheless, what do you think the velocity at $t = 2\,\text{sec}$ actually is?