WORKSHEET SOLUTIONS: THE FALLING CALCULUS BOOK

Names and student IDs: Solutions [$\pi\pi\pi$-$\pi\pi$-$\pi\pi\pi\pi$]

The gravitational constant at the surface of the planet Yuggxth is exactly $2\,\mathrm{m/sec}^2$. It has no atmosphere, so falling objects encounter no friction. A calculus book is dropped down a vertical hole at time $t = 0\,\mathrm{sec}$, so its height at time $t\,\mathrm{sec}$ is $p(t) = -t^2\,\mathrm{m}$ above the surface. (Nobody on Yuggxth knows any calculus.) Note: $-t^2$, because the calculus book is falling *down*.

We want to know the velocity of the calculus book at time $t = 2\,\mathrm{sec}$. (This is analogous to the reading of the speedometer in a car.)

As a first approximation, let's find the average velocity over the time interval $[2, 4]$ (in seconds). It is

$$\frac{p(4) - p(2)}{4 - 2} = \frac{-4^2 - (-2^2)}{4 - 2} = \frac{-12}{2} = -6\,\mathrm{m/sec}.$$

The answer is negative because the calculus book is moving down and we measured height in meters *above* the surface. Thus, the calculus book is is rising at $-6\,\mathrm{m/sec}$, or falling at $6\,\mathrm{m/sec}$. Keep the signs right!

(What happens if we measure height in meters *below* the surface?)

Use the same method to find the average velocity over the time interval $[2, 3]$ (in seconds).

Solution.

$$\frac{p(3) - p(2)}{3 - 2} = \frac{-3^2 - (-2^2)}{3 - 2} = \frac{-5}{1} = -5\,\mathrm{m/sec}.$$

□

Use the same method to find the average velocity over the time interval $[2, 2.1]$ (in seconds). (Feel free to use a calculator on this one.)

Solution.

$$\frac{p(2.1) - p(2)}{2.1 - 2} = \frac{-2.1^2 - (-2^2)}{2.1 - 2} = \frac{-0.41}{0.1} = -4.1\,\mathrm{m/sec}.$$

□

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Use the same method to find the average velocity over the time interval $[2, 2.01]$ (in seconds). (Feel free to use a calculator on this one.)

Solution.

$$\frac{p(2.01) - p(2)}{2.01 - 2} = \frac{-2.01^2 - (-2^2)}{2.01 - 2} = \frac{-0.0401}{0.01} = -4.01\,\mathrm{m/sec}.$$

□

Use the same method to find the average velocity over the time interval $[1, 2]$ (in seconds).

Solution.

$$\frac{p(1) - p(2)}{1 - 2} = \frac{-1^2 - (-2^2)}{1 - 2} = \frac{3}{-1} = -3\,\mathrm{m/sec}.$$

□

Use the same method to find the average velocity over the time interval $[1.9, 2]$ (in seconds). (Feel free to use a calculator on this one.)

Solution.

$$\frac{p(1.9) - p(2)}{1.9 - 2} = \frac{-1.9^2 - (-2^2)}{1.9 - 2} = \frac{0.39}{-0.1} = -3.9\,\mathrm{m/sec}.$$

□

What goes wrong if you try find the velocity at $t = 2\,\mathrm{sec}$ by to taking both times to be $t = 2\,\mathrm{sec}$?

Solution. You get the meaningless expression "$\frac{0}{0}$". (Important note: the expression "$\frac{0}{0}$" can **never** appear in an equation.) □

Nevertheless, what do you think the velocity at $t = 2\,\mathrm{sec}$ actually is?

Solution. Looking at the results above, it sure seems that the correct velocity at $t = 2\,\mathrm{sec}$ is $-4\,\mathrm{m/sec}$. □