WORKSHEET SOLUTIONS: DERIVATIVES FROM LINEARITY FORMULAS

Names and student IDs: Solutions [$\pi\pi\pi$-$\pi\pi$-$\pi\pi\pi\pi$]

Recall:

(1) If c is a constant, and f is the function $f(x) = c$ for all real x, then $f'(x) = 0$ for all real x.
(2) If g is the function $g(x) = x$ for all real x, then $g'(x) = 1$ for all real x. (This is a special case of rule (5) below.)
(3) If f and ~~f~~ g are differentiable at a, then $f + g$ and $f - g$ are differentiable at a, with $(f + g)'(a) = f'(a) + g'(a)$ and $(f - g)'(a) = f'(a) - g'(a)$.
(4) If f is differentiable at a, c is a constant, and k is the function $k(x) = cf(x)$, then k is differentiable at a, with $k'(a) = cf'(a)$. For short, $(cf)'(a) = cf'(a)$.
(5) If n is any positive integer, then the function $f(x) = x^n$ for all real x is differentiable everywhere, and $f'(x) = nx^{n-1}$.

In fact, the rule (5) is still correct for $x > 0$ when n is any real number, and also for $x < 0$ if $n = p/q$ for integers p and q with q odd, so that $f(x)$ is defined when $x < 0$.

1. Let f be the function $f(x) = -43$ for or all real x. Find $f'(x)$ and $f'(9)$.

Solution. By rule (1) above, $f'(x) = 0$ for all real x, so $f'(9) = 0$. □

2. Let g be the function $g(x) = x^5$ for or all real x. Find $g'(x)$ and ~~$f'(-2)$~~$g'(-2)$.

Solution. By rule (5) above, $g'(x) = 5x^4$ for all real x, so $g'(-2) = 5(-2)^4 = 5 \cdot 16 = 80$. □

3. Let f be the function $f(x) = x^7 - x^3$ for or all real x. Find $f'(x)$ and $f'(1)$.

Solution. By rules (3) and (1) above, $f'(x) = 7x^6 - 3x^2$ for all real x, so $f'(1) = 7 \cdot 1^6 - 3 \cdot 1^2 = 4$. □

4. Let q be the function $q(x) = -3x^6 - 5x^4 + \sqrt{2}$ for or all real x. Find ~~$f'(x)$ and $f'(a)$~~$q'(x)$ and $q'(a)$.

Solution. By rules (1), (3), (4), and (1) above, $q'(x) = -3 \cdot 6x^5 - 5 \cdot 4x^3 = -18x^5 - 20x^3$ for all real x, so $q'(a) = -18a^5 - 20a^3$. (Note: $\sqrt{2}$ is a constant, so its derivative is zero!) □

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5. Let f be the function $f(x) = x^2 + 4$ for or all real x, and let g be the function $g(x) = x^3 - 1$ for or all real x~~. Find $f'(x)$ and $f'(9)$~~.

Does any rule above directly apply to finding $(fg)'(x)$?

Solution. No. □

Expand $(fg)(x)$.

Solution.

$$(fg)(x) = f(x)g(x) = (x^2 + 4)(x^3 - 1) = x^5 + 4x^3 - x^2 - 4.$$

□

Find $(fg)'(x)$.

Solution. Apply the rules above to $(fg)(x) = x^5 + 4x^3 - x^2 - 4$, getting

$$(fg)'(x) = 5x^4 + 12x^2 - 2x.$$

□

What is $f'(x)g'(x)$? Is it the same as $(fg)'(x)$?

Solution. Apply the rules above to get

$$f'(x) = 2x \qquad \text{and} \qquad g'(x) = 3x^2.$$

So $f'(x)g'(x) = 6x^3$, which is certainly not the same as $(fg)'(x)$. □

The product rule, which we have not seen yet, says that $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$. Check that it give the right answer in this case.

Solution. Using the previous part,

$f'(x)g(x) + f(x)g'(x) = 2x(x^3 - 1) + (x^2 + 4) \cdot 3x^2 = 2x^4 - 2x + 3x^4 + 12x^2 = 5x^4 + 12x^2 - 2x.$

This is what you got before. □