WORKSHEET: DERIVATIVES FROM THE DEFINITION

Names and student IDs: _____

Recall: if f is a function defined on an open interval containing a, then the derivative of f at a is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

if this limit exists.

An alternate formulation is: the derivative of f at a is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a},$$

if this limit exists.

1. Let $k(x) = x^2$ for all real numbers x. Let a be an arbitrary real number. Find $k'(a)$ directly from the definition. You should get $k'(a) = 2a$.

2. If $k(x) = x^2$ for all real numbers x, then for all real numbers x, we have $k'(x) =$ ____.

3. Expand the expression $(x + h)^3$.

4. Let $l(x) = x^3$ for all real numbers x. Let a be an arbitrary real number. Find $l'(x)$ directly from the definition. You should get $l'(x) = 3x^2$.

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5. For a nonnegative integer n, let $P_n(x)$ be the function $P_n(x) = x^n$ for all real numbers x. (Take $P_0(0) = 1$.)

You have seen $P_0'(x)$, $P_1'(x)$, $P_2'(x)$, and $P_3'(x)$. What do you think $P_4'(x)$ is? What do you think $P_n'(x)$ is?