

## WORKSHEET SOLUTIONS: DERIVATIVES FROM LINEARITY FORMULAS

Names and student IDs: Solutions  $[\pi\pi\pi-\pi\pi-\pi\pi\pi\pi]$

Recall:

- (1) If  $c$  is a constant, and  $f$  is the function  $f(x) = c$  for all real  $x$ , then  $f'(x) = 0$  for all real  $x$ .
- (2) If  $g$  is the function  $g(x) = x$  for all real  $x$ , then  $g'(x) = 1$  for all real  $x$ . (This is a special case of rule (5) below.)
- (3) If  $f$  and  $g$  are differentiable at  $a$ , then  $f + g$  and  $f - g$  are differentiable at  $a$ , with  $(f + g)'(a) = f'(a) + g'(a)$  and  $(f - g)'(a) = f'(a) - g'(a)$ .
- (4) If  $f$  is differentiable at  $a$ ,  $c$  is a constant, and  $k$  is the function  $k(x) = cf(x)$ , then  $k$  is differentiable at  $a$ , with  $k'(a) = cf'(a)$ . For short,  $(cf)'(a) = cf'(a)$ .
- (5) If  $n$  is any positive integer, then the function  $f(x) = x^n$  for all real  $x$  is differentiable everywhere, and  $f'(x) = nx^{n-1}$ .

In fact, the rule (5) is still correct for  $x > 0$  when  $n$  is any real number, and also for  $x < 0$  if  $n = p/q$  for integers  $p$  and  $q$  with  $q$  odd, so that  $f(x)$  is defined when  $x < 0$ .

1. Let  $f$  be the function  $f(x) = -43$  for or all real  $x$ . Find  $f'(x)$  and  $f'(9)$ .

*Solution.* By rule (1) above,  $f'(x) = 0$  for all real  $x$ , so  $f'(9) = 0$ . □

2. Let  $g$  be the function  $g(x) = x^5$  for or all real  $x$ . Find  $g'(x)$  and  $g'(-2)$ .

*Solution.* By rule (5) above,  $g'(x) = 5x^4$  for all real  $x$ , so  $g'(-2) = 5(-2)^4 = 5 \cdot 16 = 80$ . □

3. Let  $f$  be the function  $f(x) = x^7 - x^3$  for or all real  $x$ . Find  $f'(x)$  and  $f'(1)$ .

*Solution.* By rules (3) and (1) above,  $f'(x) = 7x^6 - 3x^2$  for all real  $x$ , so  $f'(1) = 7 \cdot 1^6 - 3 \cdot 1^2 = 4$ . □

4. Let  $q$  be the function  $q(x) = -3x^6 - 5x^4 + \sqrt{2}$  for or all real  $x$ . Find  $q'(x)$  and  $q'(a)$ .

*Solution.* By rules (1), (3), (4), and (1) above,  $q'(x) = -3 \cdot 6x^5 - 5 \cdot 4x^3 = -18x^5 - 20x^3$  for all real  $x$ , so  $q'(a) = -18a^5 - 20a^3$ . (Note:  $\sqrt{2}$  is a constant, so its derivative is zero!) □

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5. Let  $f$  be the function  $f(x) = x^2 + 4$  for or all real  $x$ , and let  $g$  be the function  $g(x) = x^3 - 1$  for or all real  $x$ .

Does any rule above directly apply to finding  $(fg)'(x)$ ?

*Solution.* No. □

Expand  $(fg)(x)$ .

*Solution.*

$$(fg)(x) = f(x)g(x) = (x^2 + 4)(x^3 - 1) = x^5 + 4x^3 - x^2 - 4.$$

□

Find  $(fg)'(x)$ .

*Solution.* Apply the rules above to  $(fg)(x) = x^5 + 4x^3 - x^2 - 4$ , getting

$$(fg)'(x) = 5x^4 + 12x^2 - 2x.$$

□

What is  $f'(x)g'(x)$ ? Is it the same as  $(fg)'(x)$ ?

*Solution.* Apply the rules above to get

$$f'(x) = 2x \quad \text{and} \quad g'(x) = 3x^2.$$

So  $f'(x)g'(x) = 6x^3$ , which is certainly not the same as  $(fg)'(x)$ . □

The product rule, which we have not seen yet, says that  $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$ . Check that it give the right answer in this case.

*Solution.* Using the previous part,

$$f'(x)g(x) + f(x)g'(x) = 2x(x^3 - 1) + (x^2 + 4) \cdot 3x^2 = 2x^4 - 2x + 3x^4 + 12x^2 = 5x^4 + 12x^2 - 2x.$$

This is what you got before. □