WORKSHEET SOLUTIONS: FINDING THE ABSOLUTE MINIMUM AND MAXIMUM 1

Names and student IDs: Solutions [$\pi\pi\pi$-$\pi\pi$-$\pi\pi\pi\pi$]

Recall the method for maximizing and minimizing a continuous function f on a closed interval $[a, b]$:

(1) Find all the critical numbers c of f in $[a, b]$, that is, numbers c in $[a, b]$ such that $f'(c) = 0$ or $f'(c)$ does not exist.
(2) Evaluate f (**not** f'!) at all the numbers c found in (1) and at the endpoints a and b.
(3) The absolute maximum of f on $[a, b]$ is the largest value you got in (2). It occurs at the value of x you used to produce it. (In rare cases, the absolute maximum can occur at more than one value of x.) Similarly, the absolute minimum of f on $[a, b]$ is the smallest value you got in (2), etc.

Let $h(x) = e^{x/2}\left(164 - 73x + 7x^2\right)$. Use the methods of calculus to find the exact values of x (*not* calculator approximations) at which h has its maximum and minimum values on the interval $[-1, 5]$.

You don't yet know how to find $h'(x)$. So here it is: $h'(x) = \frac{1}{2}(7x - 3)(x - 6)e^{x/2}$.

Also, the problem does not ask you to find the exact maximum and minimum values of h on the interval, only the exact values of x at which they occur.

1. Find all critical numbers of h. (Since h' is already factored, this is easy.)

Solution. Since h is differentiable everywhere, we only need to solve $h'(c) = 0$, that is,

$$\frac{1}{2}(7c - 3)(c - 6)e^{c/2} = 0.$$

Since $e^{c/2}$ is never zero, there are exactly two solutions, namely $c = 6$ and $c = \frac{3}{7}$. □

2. Which critical numbers are in $[-1, 5]$?

Solution. Only $\frac{3}{7}$ is in $[-1, 5]$. □

3. List the values of x at which one should evaluate h according to Step (2).

Solution. Use $x = -1$, $x = 5$ (the endpoints), and $x = \frac{3}{7}$ (the critical number in $[-1, 5]$). □

4. Presumably using a calculator, determine where h takes its absolute minimum and maximum values on $[-1, 5]$.

Solution. I got

$$h(-1) \approx 147.993, \quad h\left(\tfrac{3}{7}\right) \approx 166.023, \quad \text{and} \quad h(5) \approx -316.745.$$

Therefore the absolute maximum value occurs at $x = \frac{3}{7}$ and the absolute minimum value occurs at $x = 5$. □

The value $h(6) \approx -441.882$ is smaller than any of these, but we don't use it since 6 is not in the interval $[-1, 5]$.

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