MATH 251 (PHILLIPS): SOLUTIONS TO WRITTEN HOMEWORK 3 PART 1

This homework sheet is due in class on Wednesday 22 January 2025 (week 3), in class. Write answers on a separate piece of 8.5 by 11 inch paper, well organized and well labelled, with each solution starting on the left margin of the page. Or, print this page and write on it, using the back for the second problem if needed.

All the requirements in the sheet on general instructions for homework apply. In particular, show your work (unlike WeBWorK), give exact answers (not decimal approximations), and **use correct notation.** (See the course web pages on notation.) Some of the grade will be based on correctness of notation in the work shown.

Point values as indicated, total 20 points.

1. (10 points.) This problem is about using correct notation. Accordingly, almost all the credit is for correctness of notation. See the solutions to Written Homework 2.

Consider the problem of finding the exact value of $\displaystyle\lim_{x \to -2} \frac{x^4 + x^3 - 2x^2}{x+2}$. The method is to factor the numerator and cancel one of the factors. The factors of the numerator are $x + 2$, x^2, and $x - 1$.

Write out the calculation in full, in correct notation which exhibits correctly the steps of the calculation. In particular, put "=" and "lim" everywhere they belong, and nowhere else. Start by writing $\displaystyle\lim_{x \to -2} \frac{x^4 + x^3 - 2x^2}{x+2}$. Show at least the following steps:

(1) After factoring but before cancellation.
(2) After cancellation but before substituting $x = -2$.
(3) After substituting $x = -2$ but before possible simplification.
(4) The simplified final result, if the result in the previous step can be simplified.

Solution.

$$\begin{aligned}\lim_{x \to -2} \frac{x^4 + x^3 - 2x^2}{x+2} &= \lim_{x \to -2} \frac{(x+2)x^2(x-1)}{x+2} \\ &= \lim_{x \to -2} x^2(x-1) = (-2)^2(-2-1) = 4(-3) = -12.\end{aligned}$$

This completes the solution. □

Alternate solution. For $x \neq -2$, we have

$$\frac{x^4 + x^3 - 2x^2}{x+2} = \frac{(x+2)x^2(x-1)}{x+2} = x^2(x-1).$$

Therefore

$$\lim_{x \to -2} \frac{x^4 + x^3 - 2x^2}{x+2} = \lim_{x \to -2} x^2(x-1) = (-2)^2(-2-1) = 4(-3) = -12.$$

This completes the solution. □

Comments. In the solutions above, the symbol "=" must appear in all the places where it is shown, and may not appear anywhere else.

The symbol "$\lim_{x \to -2}$" must appear in all the places where it is shown, and may not appear anywhere else. In particular,

$$\xcancel{\lim_{x \to -2} x^2(x-1) = \lim_{x \to -2} (-2)^2(-2-1)}$$

is a mathematically true statement, but does not correctly show the intended step.

Every parenthesis shown is essential. In particular, $\xcancel{-2^2}$ is wrong. (It is equal to -4, while the correct expression, $(-2)^2$, is equal to 4.) Putting in extra parentheses is not formally wrong, but should not be done because it makes the solution harder to read.

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2. (10 points.) This problem is mostly about using correct notation. Accordingly, most of the credit is for correctness of notation. See the example done in class Tuesday 21 January.

Consider the problem of finding the derivative of the function $m(t) = 2t^7 - bt^{-2} - \pi^3$, in which b is a *constant*.

Write out the calculation in full, in correct notation which exhibits correctly the steps of the calculation. In particular, put "=" and differentiation symbols everywhere they belong, and nowhere else. Start with "$m'(t)$". Use $\frac{d}{dx}$ to indicate differentiation with respect to x, $\frac{d}{dt}$ to indicate differentiation with respect to t, etc., with appropriate parentheses.

Show at least the following steps:

(1) Using the sum, difference, and constant multiple rules for derivatives.
(2) Using the power rule on each of the steps from (1), including the "-1" part.
(3) Simplification of the expression resulting from (2).

Solution. We have

$$m'(t) = 2\frac{d}{dt}\left(t^7\right) - b\frac{d}{dt}\left(t^{-2}\right) + \frac{d}{dt}\left(\pi^3\right) = 2 \cdot 7t^{7-1} - b \cdot (-2)t^{-2-1} + 0 = 14t^6 + 2bt^{-3}.$$

□

Comments. The expression π^3 is a *constant*, so its derivative is zero.

It is wrong to use "$\not\frac{d}{dx}$", since you are differentiating with respect to t.

The expression "$\not{b \cdot -2}$" is wrong: never write two operation symbols next to each other, *especially* subtraction and multiplication. See the information on notation.

No expression like

$$\cancel{14t^6 - bt^{-2} - \pi^3} \qquad \text{or} \qquad \cancel{14t^6 + 2bt^{-3} - \pi^3}$$

may appear anywhere in any correct work: either differentiate all terms or none of them. However,

$$14t^6 - \frac{d}{dt}\left(bt^{-2}\right) + \frac{d}{dt}\left(\pi^3\right)$$

is perfectly legitimate, since for every term, it is the derivative which appears.

It is acceptable, but not good, to write "$\frac{d}{dt}t^7$". It is wrong to write "$\cancel{\frac{d}{dt}2t^7}$", because this is ambiguous.