MATH 251 (PHILLIPS) QUIZ 1, 11:00 am M 27 January 2025. 20 minutes; 20 points.

NAME: SOLUTIONS Student id: $\pi\pi\pi$-$\pi\pi$-$\pi\pi\pi\pi$

Standard exam instructions apply. In particular, no calculators, no communication devices, and no notes except as $3 \times 5$ file card, written on both sides. Also, all notation must be correct, with "=", "lim", etc. everywhere they are supposed to be, and nowhere they are not supposed to be. Write answers on this page. Use the back if necessary.

---

1. (6 points.) Let $a$ be a constant. Find the derivative of the function $g(t) = 3at^{10} - \dfrac{7}{\sqrt[4]{t}} + \pi^6$. Show at least one intermediate step. This problem is not mostly about notation, but **notation counts.**

**Solution:** The following solution shows several steps.

$$g'(t) = \frac{d}{dt}(3at^{10}) - \frac{d}{dt}\left(\frac{7}{\sqrt[4]{t}}\right) + \frac{d}{dt}(\pi^6) = 3a \cdot 10t^9 - \frac{d}{dt}(7t^{-1/4}) + 0$$
$$= 30at^9 - 7\left(-\frac{1}{4}\right)t^{-5/4} = 30at^9 + \frac{7}{4}t^{-5/4}.$$

The expression $\pi^6$ is a *constant*, so its derivative is zero.

**Alternate solution:** Rewrite

$$g(t) = 3at^{10} - 7t^{-1/4} + \pi^6.$$

Then

$$g'(t) = 3a \cdot 10t^9 - 7\left(-\frac{1}{4}\right)t^{-5/4} - 0 = 30at^9 + \frac{7}{4}t^{-5/4}.$$

2. (6 points.) Find the derivative of the function $w(x) = 31 - (x^3 - 2x)\sin(x)$. Show at least one intermediate step. This problem is not mostly about notation, but **notation counts.**

**Solution:** The following solution shows several steps. Use the product rule on the second term:

$$w'(x) = \frac{d}{dx}(31) - \frac{d}{dx}\big((x^3 - 2x)\sin(x)\big)$$
$$= 0 - \left(\frac{d}{dx}(x^3 - 2x) \cdot \sin(x) + (x^3 - 2x)\sin'(x)\right)$$
$$= -\big[(3x^2 - 2)\sin(x) + (x^3 - 2x)\cos(x)\big] = -(3x^2 - 2)\sin(x) - (x^3 - 2x)\cos(x).$$

Don't forget to distribute the minus sign across the parentheses!

3. (8 points.) This problem is about using correct notation. Accordingly, almost all the credit is for correctness of notation.

Consider the problem of finding the exact value of $\lim_{x \to -3} \dfrac{x^3 + 3x^2 + 2x + 6}{x + 3}$. The method is to factor the numerator and cancel one of the factors. The factors of the numerator are $x + 3$ and $x^2 + 2$.

Write out the calculation in full, in correct notation which exhibits correctly the steps of the calculation. In particular, put "=" and "lim" everywhere they belong, and nowhere else. Start by writing $\lim_{x \to -3} \dfrac{x^3 + 3x^2 + 2x + 6}{x + 3}$. Show at least the following steps: after factoring but before cancellation; after cancellation but before substituting $x = -3$; after substituting $x = -3$ but before possible simplification; the simplified final result, if the result in the previous step can be simplified.

**Solution:**

$$\begin{aligned} \lim_{x \to -3} \frac{x^3 + 3x^2 + 2x + 6}{x + 3} &= \lim_{x \to -3} \frac{(x + 3)(x^2 + 2)}{x + 3} \\ &= \lim_{x \to -3} (x^2 + 2) = (-3)^2 + 2 = 9 + 2 = 11. \end{aligned}$$

This completes the solution.

**Alternate solution:** For $x \neq -3$, we have

$$\frac{x^3 + 3x^2 + 2x + 6}{x + 3} = \frac{(x + 3)(x^2 + 2)}{x + 3} = x^2 + 2.$$

Therefore

$$\lim_{x \to -3} \frac{x^3 + 3x^2 + 2x + 6}{x + 3} = \lim_{x \to -3} (x^2 + 2) = (-3)^2 + 2 = 9 + 2 = 11.$$

This completes the solution.

**Comments.** In the solutions above, the symbol "=" must appear in all the places where it is shown, and may not appear anywhere else.

The symbol "$\lim_{x \to -3}$" must appear in all the places where it is shown, and may not appear anywhere else. In particular,

$$\lim_{x \to -3} (x^2 + 2) = \lim_{x \to -3} [(-3)^2 + 2]$$

is a mathematically true statement, but does not correctly show the intended step.

Every parenthesis shown is essential. In particular, writing $-3^2$ is wrong: $-3^2 = -9$ but $(-3)^2 = 9$. Putting in extra parentheses is not formally wrong, but should not be done because it makes the solution harder to read.