WORKSHEET: CHAIN RULE

Names and student IDs: ____

Old differentiation rules are on the back.

Chain rule: If g is differentiable at x and f is differentiable at $g(x)$, and if

$$h(x) = f(g(x))$$

for all x (in a suitable open interval), then

$$h'(x) = f'(g(x)) \cdot g'(x).$$

Example for the chain rule: If $h(x) = \sin(x^3)$ then write $h(x) = f(g(x))$ with

$$f(x) = \sin(x) \qquad \text{and} \qquad g(x) = x^3.$$

Thus

$$h'(x) = f'(g(x)) \cdot g'(x) = \sin'(x^3) \cdot \frac{d}{dx}\big(x^3\big) = \cos(x^3) \cdot 3x^2 = 3x^2 \cos(x^3).$$

The last step is conventional. Caution: "$\sin'(x^3)$" means you take the derivative of the sine function and evaluate it at x^3. It does *not* mean $\cancel{\frac{d}{dx}\big(\sin(x^3)\big)}$.

Now differentiate the following functions, or else tell me that no differentiation rule you have seen so far applies.

Let $w(x) = \cos(x^4)$. First, if we want to usefully write $w(x) = f(g(x))$, then

$f(x) =$ and $g(x) =$

Now, if $w(x) = \cos(x^4)$ then $w'(x) =$

Let $p(x) = (x^{11} + 3x + 1)^{109}$. First, if we want to usefully write $p(x) = f(g(x))$, then

$f(x) =$ and $g(x) =$

Now, $\frac{d}{dx}\big((x^{11} + 3x + 1)^{109}\big) =$

If $r(x) = \big(\sin(x) + 29\big)^{1/3}$, then $r'(x) =$

If f is a function such that $f'(s) = e^{s^2}$ for all real s, find $\frac{d}{dx}\big(f(\tan(x))\big)$ and $\frac{d}{dx}\big(\tan(f(x))\big)$. (Recall that $\tan'(x) = \sec^2(x)$.)

$\frac{d}{dx}\big(x^{\sin(x)}\big) =$

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Differentiation rules we have already seen:

(1) The derivative of a constant function is the zero function.

(2) $\dfrac{d}{dx}(x) = 1$.

(3) If n is a constant, then $\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ (with a suitable domain restriction if n is not a positive integer).

(4) If f and g are differentiable, and $h(x) = f(x) + g(x)$ for all x (in a suitable open interval), then $h'(x) = f'(x) + g'(x)$.

(5) If c is a constant, f is differentiable, and $h(x) = cf(x)$ for all x (in a suitable open interval), then $h'(x) = cf'(x)$.

(6) If f and g are differentiable, and $h(x) = f(x)g(x)$ for all x (in a suitable open interval), then $h'(x) = f'(x)g(x) + f(x)g'(x)$.

(7) If f and g are differentiable, $g(x)$ is never zero (on a suitable open interval) and

$$h(x) = \frac{f(x)}{g(x)}$$

for all x (in a suitable open interval), then

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}.$$