WORKSHEET SOLUTIONS: EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Names and student IDs: Solutions [$\pi\pi\pi$-$\pi\pi$-$\pi\pi\pi\pi$]

Recall the chain rule: If g is differentiable at x and f is differentiable at $g(x)$, and if $h(x) = f(g(x))$ for all x (in a suitable open interval), then

$$h'(x) = f'(g(x)) \cdot g'(x).$$

Also,

$$\frac{d}{dx}\big(e^x\big) = e^x \qquad \text{and} \qquad \frac{d}{dx}\big(\ln(x)\big) = \frac{1}{x}.$$

First, two problems related to Quiz 1:

1. Write $1/\sqrt[7]{y}$ as y^a for some a. (Use two steps if needed.)

Solution.

$$\frac{1}{\sqrt[7]{y}} = \frac{1}{y^{1/7}} = y^{-1/7}.$$

□

2. What is -5^2?

Solution. $-5^2 = -25$ (**not** $\cancel{25}$, which is $(-5)^2$). Remember the order of operations! □

Now differentiate and simplify the following functions, or else tell me that no differentiation rule you have seen so far applies:

3. $f(x) = x\ln(x) - x$.

Solution. Use the product rule: $f'(x) = \dfrac{d}{dx}\big(x\big)\ln(x) + x\ln'(x) - 1 = \ln(x) + x\big(\frac{1}{x}\big) - 1 = \ln(x)$. □

4. $g(x) = e^{x^2+7x}$.

Solution. Use the chain rule. Write $e^y = \exp(y)$ for clarity. (This is standard notation.) So $g(x) = \exp(x^2 + 7x)$ and

$$g'(x) = \exp'(x^2+7x)\frac{d}{dx}\big(x^2+7x\big) = \exp(x^2+7x)(2x+7) = (2x+7)e^{x^2+7x}.$$

Note: $\cancel{\exp(x^2+7x)2x+7}$ is wrong, because of missing parentheses. □

5. $q(x) = \ln(x^2 + e^x)$.

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Solution. Use the chain rule:

$$q'(x) = \ln'(x^2 + e^x)\frac{d}{dx}\big(x^2 + e^x\big) = \left(\frac{1}{x^2 + e^x}\right)(2x + e^x) = \frac{2x + e^x}{x^2 + e^x}.$$

This expression can't be further simplified. (See the Midterm Zero problems.) □

6. $s(x) = e^{x^2 \sin(x)}$.

Solution. Use the chain rule and the product rule. Write $e^y = \exp(y)$. Then

$$\begin{aligned} s'(x) &= \exp'(x^2 \sin(x))\frac{d}{dx}\big(x^2 \sin(x)\big) \\ &= \exp(x^2 \sin(x))\big(2x \sin(x) + x^2 \cos(x)\big) = e^{x^2 \sin(x)}\big(2x \sin(x) + x^2 \cos(x)\big). \end{aligned}$$

□

7. $p(x) = 7^x$.

Solution. No rule we have seen applies. (Not the power rule: that is for functions like x^7.)

It can, however, by done. Write $7 = e^{\ln(7)}$, so

$$7^x = \big(e^{\ln(7)}\big)^x = e^{\ln(7)x} = \exp\big(\ln(7)x\big).$$

Now use the chain rule:

$$p'(x) = \exp'\big(\ln(7)x\big)\frac{d}{dx}\big(\ln(7)x\big) = \exp\big(\ln(7)x\big) \cdot \ln(7) = \ln(7) \cdot 7^x.$$

□