WORKSHEET: INVERSE FUNCTIONS

Names and student IDs: ____

Recall the chain rule: If g is differentiable at x and f is differentiable at $g(x)$, and if $h(x) = f(g(x))$ for all x (in a suitable open interval), then

$$h'(x) = f'(g(x)) \cdot g'(x).$$

You will also need $\tan'(x) = \sec^2(x)$.

1. First, just an example. Differentiate the function $q(x) = \arcsin(e^{-x})$, which is defined and differentiable for $x > 0$.

Next, let's find $\arctan'(x)$, from "scratch".

2. Is it more useful to differentiate both sides of the equation of functions $\arctan(\tan(x)) = x$ (valid when $-\frac{\pi}{2} < x < \frac{\pi}{2}$) or $\tan(\arctan(x)) = x$ (valid for all real x)? Remember that you will use the chain rule, and you want $\arctan'(x)$ somewhere in the answer.

3. Carry out the differentiation from the previous step, and solve for $\arctan'(x)$.

4. Use a trigonometric identity to eliminate all trigonometric functions in the previous answer. (The identity is less commonly used than the one needed for $\arcsin'(x)$, but the other steps are less complicated.)

5. Now repeat for the inverse function Q (defined for all real x) of the function $h(x) = x^7 + x + 6$. You won't be able to simplify the way we did with $\arctan'(x)$).

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