WORKSHEET: IMPLICIT DIFFERENTIATION

Names and student IDs: ____

Recall the chain rule: If g is differentiable at x and f is differentiable at $g(x)$, and if $h(x) = f(g(x))$ for all x (in a suitable open interval), then

$$h'(x) = f'(g(x)) \cdot g'(x).$$

1. Use implicit differentiation to find $\frac{dy}{dx}$: $x^2 + y^2 = 49$. You **must** solve for $\frac{dy}{dx}$.

The above is how these problems are written in the textbook and in many WeBWorK problems. It doesn't really tell you what it should, but you need to get used to it. (Sorry: I can't change the textbook or what physicists do.) It means the following.

Assume y is a differentiable function of x given implicitly by $x^2 + [y(x)]^2 = 49$. Find $y'(x)$ in terms of x and $y(x)$.

2. If $y^3 = \sin(11x - y) - \sin(7)$, find $\frac{dy}{dx}$ by implicit differentiation. (You must solve for $\frac{dy}{dx}$.)

Next, back to $\arctan'(x)$, from "scratch".

3. Is it more useful to differentiate both sides of the equation of functions $\arctan(\tan(x)) = x$ (valid when $-\frac{\pi}{2} < x < \frac{\pi}{2}$) or $\tan(\arctan(x)) = x$ (valid for all real x)? Remember that you will use the chain rule, and you want $\arctan'(x)$ somewhere in the answer.

4. Carry out the differentiation from the previous step, and solve for $\arctan'(x)$.

Continued on back.

Date: 4 February 2024.

5. Use a trigonometric identity to eliminate all trigonometric functions in the previous answer. (The identity is less commonly used than the one needed for $\arcsin'(x)$, but the other steps are less complicated.)

5. Now repeat for the inverse function Q (defined for all real x) of the function $h(x) = x^7 + x + 6$. You won't be able to simplify the way we did with $\arctan'(x)$).