WORKSHEET SOLUTIONS: IMPLICIT DIFFERENTIATION 2

Names and student IDs: Solutions [$\pi\pi\pi$-$\pi\pi$-$\pi\pi\pi\pi$]

Recall the chain rule: If g is differentiable at x and f is differentiable at $g(x)$, and if $h(x) = f(g(x))$ for all x (in a suitable open interval), then

$$h'(x) = f'(g(x)) \cdot g'(x).$$

Further reminders: in implicit differentiation problems, y (or some other variable) is implicitly a function of x (or some other variable). So, for example, $\frac{d}{dx}\left(y^3\right) = 3y^2\frac{dy}{dx}$, not zero (and certainly not $3\cancel{y^2}$—that is **never** right).

Also, $\frac{dy}{dx}(x^2y + y^6)$ means the product of $\frac{dy}{dx}$ and $x^2y + y^6$. It does **not** mean the derivative of $x^2y + y^6$ with respect to x. That is correctly written $\frac{d}{dx}\left(x^2y + y^6\right)$. Getting this wrong is serious error.

You will use implicit differentiation to find $\frac{dy}{dx}$ when $y^7 = \tan(3x - y) + \pi^3$. You **must** solve for $\frac{dy}{dx}$.

1. Rewrite the formula with y written as a function of x.

Solution: $y(x)^7 = \tan(3x - y(x)) + \pi^3$.

2. There are two places you will need the chain rule. What are they?

Solution: When differentiating y^7 (or $y(x)^7$), and when differentiating $\tan(3x - y)$ (or $\tan(3x - y(x))$).

3. Carry out the implicit differentiation. (You must solve for $\frac{dy}{dx}$.)

Solution written using $y'(x)$: We have:

$$y(x)^7 = \tan(3x - y(x)) + \pi^3.$$

Differentiate both sides with respect to x, using the chain rule on both sides:

$$7[y(x)]^6y'(x) = \sec^2(3x - y(x))\frac{d}{dx}(3x - y(x)) = \sec^2(3x - y(x))\left(3 - y'(x)\right).$$

(The derivative of π^3 is zero because π^3 is a constant.)

Now solve for $y'(x)$:

$$7[y(x)]^6y'(x) = 3\sec^2(3x - y(x)) - \sec^2(3x - y(x))y'(x)$$
$$7[y(x)]^6y'(x) + \sec^2(3x - y(x))y'(x) = 3\sec^2(3x - y(x))$$

Date: 7 February 2024.

$$y'(x) = \frac{3\sec^2(3x - y(x))}{7[y(x)]^6 + \sec^2(3x - y(x))}.$$

This expression can't be further simplified.

Solution written using $\frac{dy}{dx}$*:* Differentiate with respect to x, using the chain rule on both sides, just as before:

$$7y^6\frac{dy}{dx} = \sec^2(3x - y)\frac{d}{dx}(3x - y) = \sec^2(3x - y)\left(3 - \frac{dy}{dx}\right).$$

(The derivative of π^3 is zero because π^3 is a constant.)

Now solve for $\frac{dy}{dx}$:

$$\begin{aligned} 7y^6\frac{dy}{dx} &= 3\sec^2(3x - y) - \sec^2(3x - y)\frac{dy}{dx} \\ 7y^6\frac{dy}{dx} + \sec^2(3x - y)\frac{dy}{dx} &= 3\sec^2(3x - y) \\ \frac{dy}{dx} &= \frac{3\sec^2(3x - y)}{7y^6 + \sec^2(3x - y)}. \end{aligned}$$

As before, this expression can't be further simplified.