

MATH 251 (PHILLIPS): WRITTEN HOMEWORK 5

This homework sheet is due in class on Wednesday 5 February 2025 (week 5), in class. Write answers on a separate piece of 8.5 by 11 inch paper, well organized and well labelled, with each solution starting on the left margin of the page. Or, print a 2-sides copy of this page and write on it.

All the requirements in the sheet on general instructions for homework apply. In particular, show your work (unlike WeBWorK), give exact answers (not decimal approximations), and **use correct notation**. (See the course web pages on notation.) Some of the grade will be based on correctness of notation in the work shown.

Point values as indicated, total 50 points.

1. (20 points.) A postmodern artist wants to make a frame for a painting consisting of four different exotic materials on the four sides. The material for the top costs 1 shekel per inch, the material for the bottom costs 3 shekels per inch, the material for the right hand side costs 7 shekels per inch, and the material for the left hand side costs 9 shekels per inch. The frame is to cost at most 320 shekels. Use calculus to find the width and height of the painting of largest area that can be enclosed. Include units. Verify that your solution corresponds to the largest possible area, using methods we have seen so far in the course.

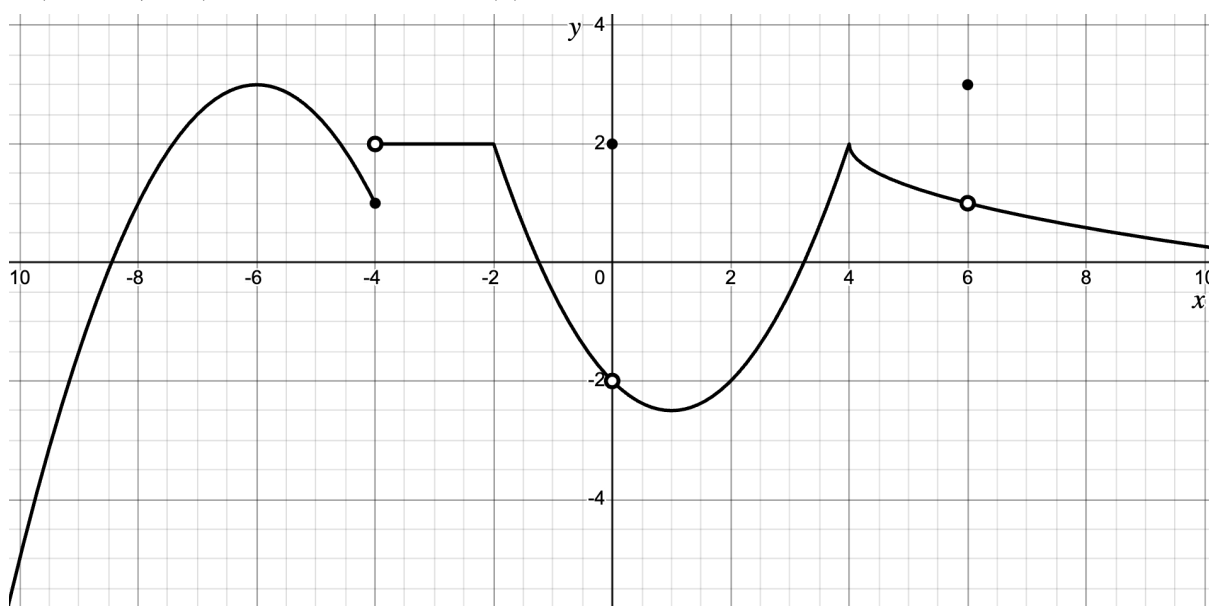
(Problem 2 is on the back.)

Date: 31 January 2025.

For previous examples for Problem 2 below, see the solutions to the real Midterm 1 for 9:00 am (https://pages.uoregon.edu/ncp/Courses/Math251_W25_Web/Midterm_1/M1_251_W2025_09_Soln.pdf) (Problem 10), the solutions to the real Midterm 1 for 11:00 am (https://pages.uoregon.edu/ncp/Courses/Math251_W25_Web/Midterm_1/M1_251_W2025_11.pdf) (Problem 10), the solutions to the Midterm 1 review session worksheet (https://pages.uoregon.edu/ncp/Courses/Math251_W25_Web/Midterm_1/M1Worksheet_Soln.pdf) (Problem 10), the solutions to the sample Midterm 1 (https://pages.uoregon.edu/ncp/Courses/Math251_W25_Web/Midterm_1/M1Sample_Soln.pdf) (Problems 8 and 9), and the solutions to Written Homework 2 (https://pages.uoregon.edu/ncp/Courses/Math251_W25_Web/Weekly_schedule/Week_02/WrittenHW2_Soln.pdf) (Problem 5). In the textbook, see the graphs on the following pages, and the associated discussion: 135, 140, 143, 145 (for now, the important point here is that $\lim_{x \rightarrow 2} f(x)$ does not exist), 151, 180, 182, 235–237, and 239. (We will return to one sided limits and to infinite limits.)

It is very important that you understand limits, continuity, and derivatives in terms of the pictures. Otherwise, the pictures I draw on the board during class etc. will not make sense.

2. (5 points/part) For the function $y = w(x)$ graphed below, answer the following questions.



- Does $\lim_{x \rightarrow -4} w(x)$ exist? If so, what is it? If not, why not?
- Does $\lim_{x \rightarrow 6} w(x)$ exist? If so, what is it? If not, why not?
- What is the largest interval containing 2 on which w is continuous? Why?
- Which of the following best describes $w'(-9)$? Why?
 - $w'(-9)$ does not exist.
 - $w'(-9)$ is close to 0.
 - $w'(-9)$ is positive and not close to 0.
 - $w'(-9)$ is negative and not close to 0.
 - None of the above.
- Which of the following best describes $w'(-6)$? Why?
 - $w'(-6)$ does not exist.
 - $w'(-6)$ is close to 0.
 - $w'(-6)$ is positive and not close to 0.
 - $w'(-6)$ is negative and not close to 0.
 - None of the above.
- List all points in $(-10, 10)$ at which w is not differentiable. Give reasons.