WORKSHEET: INFINITE LIMITS

Names and student IDs: ____

Recall: We say $\lim_{x\to a} f(x) = \infty$ (even though ∞ is not a number) if one can force $f(x)$ to be as large as one wants by requiring that x be close enough to a. In particular, f has a vertical asymptote at $x = a$, near which the graph goes up on both sides. Example: $\lim_{x\to o} \frac{1}{x^2} = \infty$.

Plot this on your calculator to see, moving the screen around to show the parts of the function near $x = 0$.

Also look at the following table of values. You see that when x is close to zero (with either $x > 0$ or $x < 0$), then x^2 is positive and close to zero, and $\frac{1}{x^2}$ is ;positive and large.

x	x^2	$\frac{1}{x^2}$
1	1	1
0.1	0.01	100
0.01	0.0001	10,000
0.0001	0.00000001	100,000,000
−1	1	1
−0.1	0.01	100
−0.01	0.0001	10,000
−0.0001	0.00000001	100,000,000

Similar considerations give meanings to $\lim_{x\to a} f(x) = -\infty$, $\lim_{x\to a^-} f(x) = \pm\infty$, $\lim_{x\to \infty} f(x) = \pm\infty$, etc.

1. What are the following? Try computing some values if needed. After you have answered, look at calculator graphs to check.

$\lim_{x\to 0} \left(-\frac{1}{x^2}\right)$? (Parentheses are needed in this expression.)

$\lim_{x\to 0^-} \frac{1}{x}$?

$\lim_{x\to 2^+} \frac{3}{x-2}$?

$\lim_{x\to -\infty} (7x^2 + 1)$?

Recall that ∞ and $-\infty$ are not numbers, so we can't write arithmetic operations on them. Nevertheless, some limits behave as if you could. For example, $\lim_{x\to 0} \left(-\frac{1}{x^2}\right) = -\infty$ and $\lim_{x\to 0}(x^2 - 2\cos(x)) = -2$, so $\lim_{x\to 0} \left(-\frac{1}{x^2} + x^2 - 2\cos(x)\right) = -\infty$ and $\lim_{x\to 0} \left(-\frac{1}{x^2}\right)(x^2 - 2\cos(x)) = \infty$. Reasons: when x is close to 0 but $x \neq 0$, then $-\frac{1}{x^2}$ is very far from zero and **negative**, while $x^2 - 2\cos(x)$ is very close to -2. So $-\frac{1}{x^2} + x^2 - 2\cos(x)$ is still very far from zero and **negative**, while $\frac{1}{x^2}(x^2 - 2\cos(x))$ is now very far from zero and **positive**.

Continued on back.

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2. Suppose now

$$\lim_{x\to 3} f(x) = \infty, \qquad \lim_{x\to 3} g(x) = \infty, \qquad \lim_{x\to 3} h(x) = 25, \qquad \lim_{x\to 3} j(x) = 0. \qquad \text{and} \qquad \lim_{x\to 3} k(x) = \frac{1}{100}.$$

What can you say about the following?

$\lim_{x\to 3}(f(x) + g(x))$?

$\lim_{x\to 3}(f(x) - g(x))$?

$\lim_{x\to 3}(f(x) - k(x))$?

$\lim_{x\to 3} f(x)g(x)$?

$\lim_{x\to 3} f(x)k(x)$?

$\lim_{x\to 3} f(x)j(x)$?

3. Use these ideas to find

$$\lim_{x\to 0^+}\left(\frac{26}{x} + 42x^2 + 7\right), \qquad \lim_{x\to 0^-}\left(\frac{26}{x} + 42x^2 + 7\right),$$

$$\lim_{x\to\infty}\left(\frac{26}{x} + 42x^2 + 7\right), \qquad \text{and} \qquad \lim_{x\to-\infty}\left(\frac{26}{x} + 42x^2 + 7\right).$$

4. Find $\lim_{x\to 3^+}\frac{x-7}{x-3}$.