MATH 251 (PHILLIPS): SOLUTIONS TO WRITTEN HOMEWORK 6 PART 1

This homework sheet is due in class on Monday 10 February 2025 (week 6), in class. Write answers on a separate piece of 8.5 by 11 inch paper, well organized and well labelled, with each solution starting on the left margin of the page. Or, print a 2-sides copy of this page and write on it.

All the requirements in the sheet on general instructions for homework apply. In particular, show your work (unlike WeBWorK), give exact answers (not decimal approximations), and **use correct notation.** (See the course web pages on notation.) Some of the grade will be based on correctness of notation in the work shown.

Point values as indicated, total 30 points.

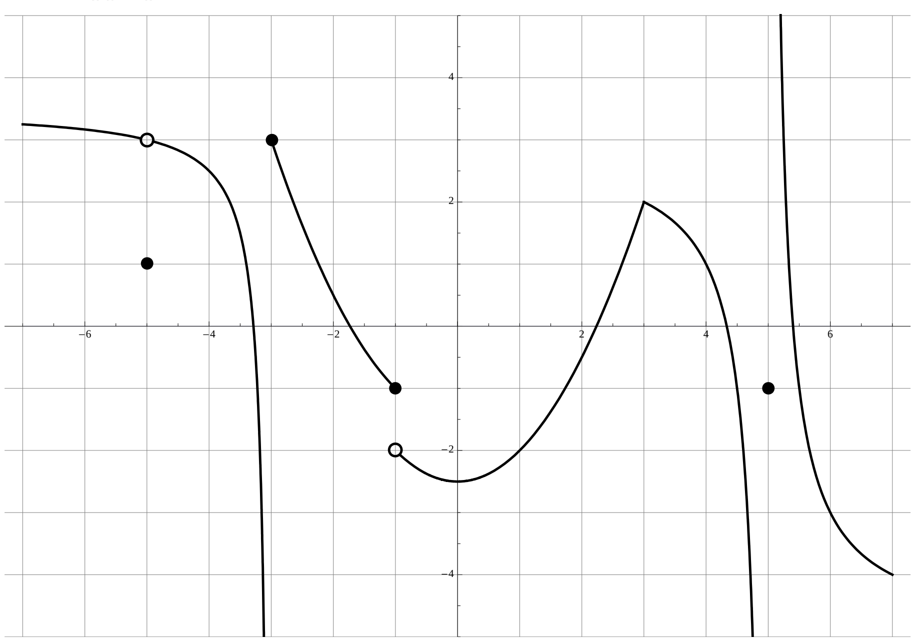
1. (15 points.) If $f(x) = \dfrac{1}{x-2}$, compute the derivative $f'(3)$ *directly from the definition*. (For one point, check your answer using the differentiation formula, but no other credit will be given for just using the formula.)

Solution: We find the limit of the difference quotient. To handle the expression that appears in the difference quotient, we subtract the fractions in the numerator and then cancel common factors in the numerator and denominator:

$$f'(3) = \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = \lim_{x \to 3} \frac{\frac{1}{x-2} - 1}{x - 3} = \lim_{x \to 3} \frac{\frac{1}{x-2} - \frac{x-2}{x-2}}{x - 3}$$
$$= \lim_{x \to 3} \frac{\left(\frac{3-x}{x-2}\right)}{x - 3} = \lim_{x \to 3} \left(\frac{3-x}{x-2} \cdot \frac{1}{x-3} \right) = \lim_{x \to 3} \frac{-1}{x-2} = -1.$$

We can check using the differentiation formulas. It is easiest to use the chain rule from Section 3.4: $f(x) = (x-3)^{-1}$, so $f'(x) = -(x-3)^{-2} \cdot (1) = -(x-3)^{-2}$, whence $f'(3) = -1$. It can also be done using the quotient rule. (However, you get only one point if this is the only thing you do.)

2. (5 points/part) For the function $y = g(x)$ graphed below, answer the following questions about limits, using ∞ or $-\infty$ when appropriate:



Date: 10 February 2025.

(a) Does $\lim_{x \to 5^+} g(x)$ exist, or is it ∞ or $-\infty$? If so, what is it, or which of ∞ or $-\infty$ is it? If not, why not?

Solution: You can see from the graph that one can make $g(x)$ as large as one wants by requiring that x be close enough to 5 but greater than -5. Therefore $\lim_{x \to 5^+} g(x) = \infty$.

It is not true that ~~$\lim_{x \to 5^+} g(x)$~~ $= -1$. That is $g(5)$.

(b) Does $\lim_{x \to -3} g(x)$ exist, or is it ∞ or $-\infty$? If so, what is it, or which of ∞ or $-\infty$ is it? If not, why not?

Solution: The limit does not exist, and is also neither ∞ nor $-\infty$, because $\lim_{x \to -3^-} g(x) = \infty$ but $\lim_{x \to -3^+} g(x) = 3$, which is different.

(c) Is g continuous at $x = 3$? Why or why not?

Solution: Yes, g continuous at $x = 3$, since $\lim_{x \to 3} g(x) = 2 = g(3)$.