WORKSHEET SOLUTIONS: DERIVATIVES AND LOCAL EXTREMUMS

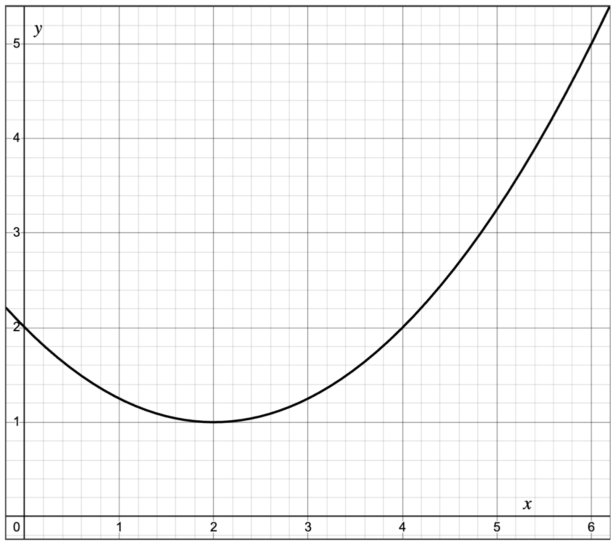
Names and student IDs: Solutions [$\pi\pi\pi$-$\pi\pi$-$\pi\pi\pi\pi$]

Recall that a differentiable function f is increasing on (a,b) exactly when $f'(x) \geq 0$ on (a,b), and that f is decreasing on (a,b) exactly when $f'(x) \leq 0$ on (a,b).

Also recall that $f'(a)$ is the slope of the tangent line to the graph of $y = f(x)$ at $x = a$ (the point $(a, f(a))$ on the graph).

1. Draw the graph of a differentiable (in particular, continuous) function f which is decreasing on the open interval $(0,2)$ and increasing on the open interval $(2,6)$. What kind of feature does this function have at $x = 2$?

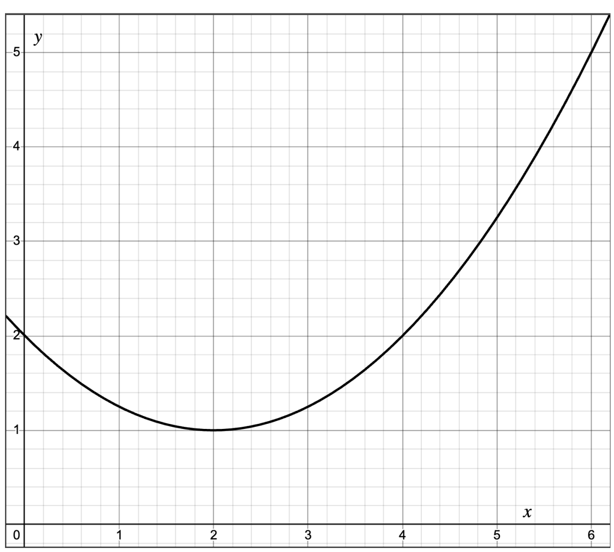
Here is one such graph (of many possibilities):



This function has a local minimum at $x = 2$.

2. Draw the graph of a differentiable (in particular, continuous) function f such that $f'(x) < 0$ on the open interval $(0,2)$ and $f'(x) > 0$ on the open interval $(2,6)$. What kind of feature does this function have at $x = 2$?

Here is one such graph (of many possibilities):



This function has a local minimum at $x = 2$.

3. Draw the graph of a differentiable (in particular, continuous) function f such that $f'(3) = 0$ and $f'(x)$ is increasing on the interval $[3,6)$. In particular, as you move right through $[3,6)$, the slopes of the tangent lines are positive and the tangent lines are getting steeper.

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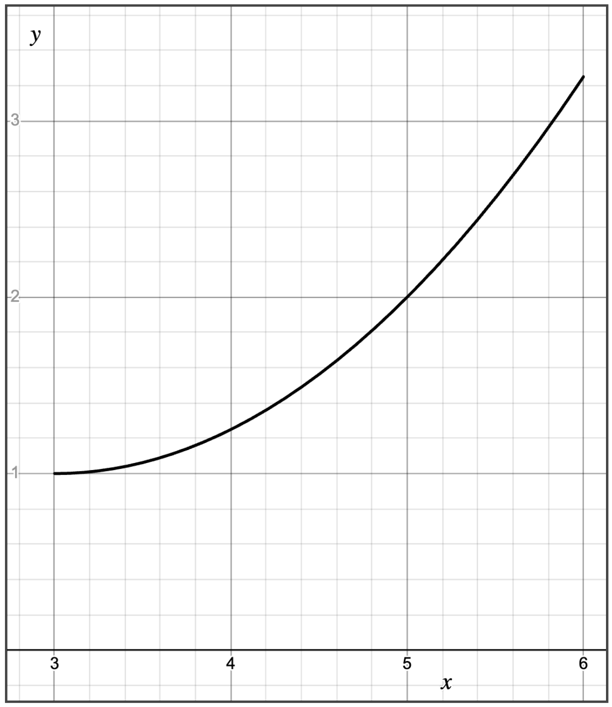
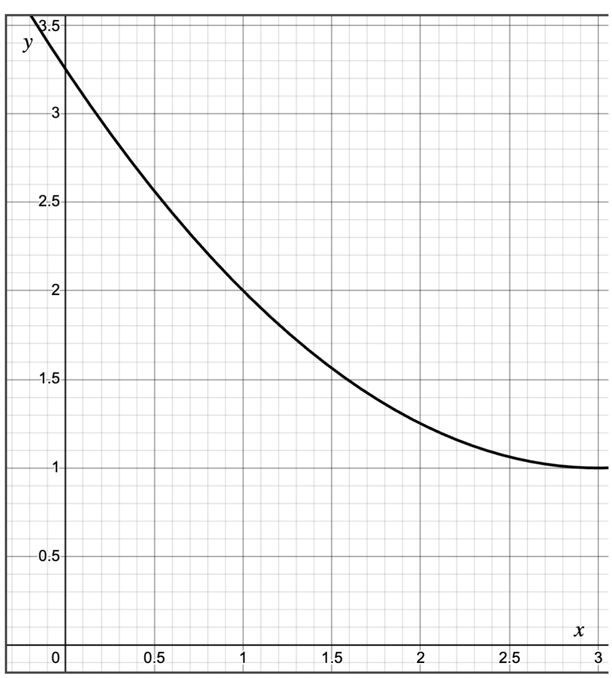
Here is one such graph (of many possibilities):



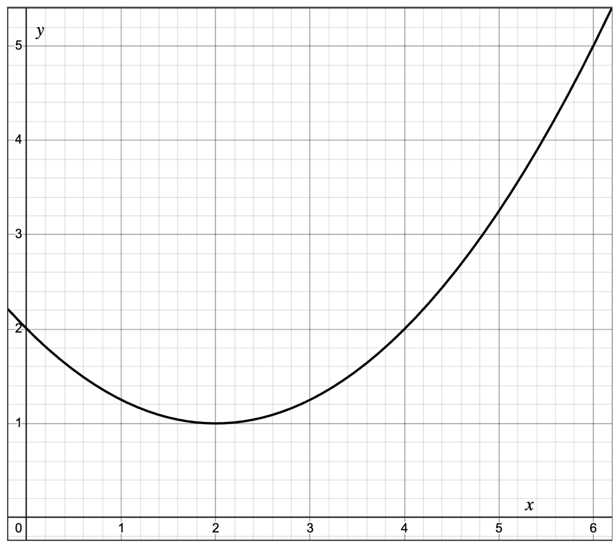
4. Draw the graph of a differentiable (in particular, continuous) function f such that $f'(3) = 0$ and $f'(x)$ is increasing on the interval $(0, 3]$. In particular, as you move right through $(0, 3]$, the slopes of the tangent lines are negative and the tangent lines are getting less steep (since, for example, $-2 < -1 < 0$).

Here is one such graph (of many possibilities):



5. Draw the graph of a differentiable (in particular, continuous) function f such that $f'(2) = 0$ and $f'(x)$ is increasing on the open interval $(0, 6)$. In particular, as you move right through $(0, 6)$, the slopes of the tangent lines are increasing and switch from negative to positive at $x = 2$. What kind of feature does this function have at $x = 2$?

Here is one such graph (of many possibilities):

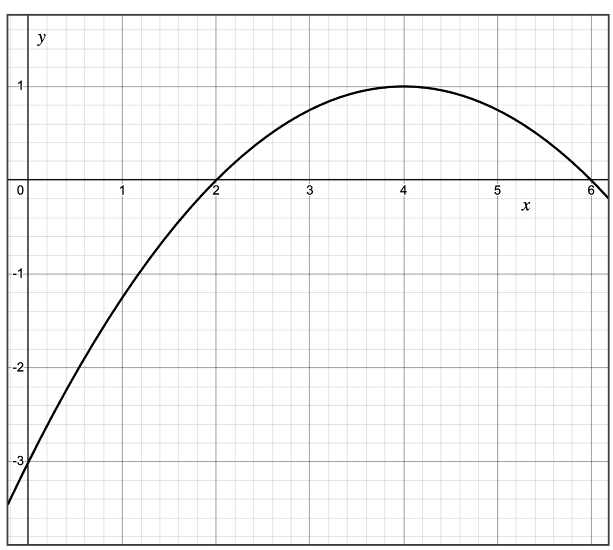


This function has a local minimum at $x = 2$.

6. From the reminder at the top of the page, but applied with $f = g'$: if g' is differentiable, then g' is decreasing on (a, b) exactly when $g''(x) \leq 0$ on (a, b).

Draw the graph of a twice differentiable (in particular, continuous) function g such that $g'(4) = 0$ and $g''(x) < 0$ on the open interval $(0, 6)$. What kind of feature does this function have at $x = 2$?

Here is one such graph (of many possibilities):



This function has a local maximum at $x = 4$.