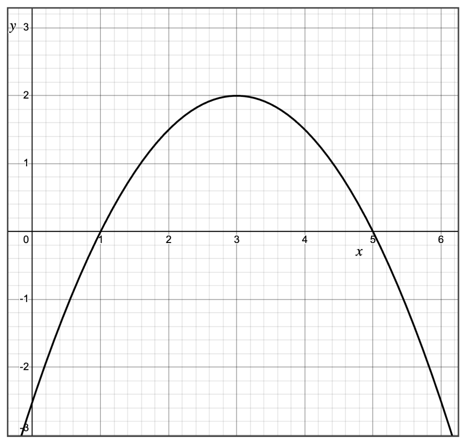
WORKSHEET: DERIVATIVES AND LOCAL EXTREMUMS

Names and student IDs: ____

1. Shown below is the graph of $y = h(x)$ for some differentiable function h.



Recall that $h'(c)$ is the slope of the tangent line to the graph of $y = h(x)$ at $x = c$ (the point $(c, h(c))$ on the graph).

Estimate $h'(1), h'(2), \ldots, h'(5)$ (values of the derivative of h). (If you need to, draw tangent lines on the graph and estimate their slopes.)

$$h'(1) \approx \_\_\_\_, \qquad h'(2) \approx \_\_\_\_, \qquad h'(3) \approx \_\_\_\_, \qquad h'(4) \approx \_\_\_\_, \qquad h'(5) \approx \_\_\_\_.$$

2. Using the values you found, draw a graph of $y = h'(x)$, the **derivative** of $h(x)$.

3. Is the **derivative** $h'(x)$ of $h(x)$ is increasing on the interval $(0, 6)$, decreasing on $(0, 6)$, or increasing on parts of this interval and decreasing on other parts?

(Continued on back.)

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4. Recall that a differentiable function f is increasing on an open interval (a, b) exactly when $f'(x) \geq 0$ on (a, b), and that f is decreasing on (a, b) exactly when $f'(x) \leq 0$ on (a, b).

Apply this with $f = h'$, and decide whether the **second derivative** $h''(x)$ of $h(x)$ is positive on the interval $(0, 6)$, negative on $(0, 6)$, or positive on parts of this interval and negative on other parts.

5. Draw the graph of some function $y = k(x)$ on the interval $(0, 6)$ such that $(3) = 0$ and $h''(x) > 0$ on all of $(0, 6)$. Does your function have a local minimum or a local maximum at $x = 3$? Is this function concave up or concave down on $(0, 6)$?

6. Let $g(x) = x^3 - 6x^2$. By considering the signs of $g'(x)$ and $g''(x)$, find the following:

Critical points of g:

On which open intervals is g increasing?

On which open intervals is g decreasing?

Local maximums of g:

Local minimums of g:

On which open intervals is g concave up?

On which open intervals is g concave down?