MATH 251 (PHILLIPS): SOLUTIONS TO WRITTEN HOMEWORK 6 PART 2

This homework sheet is due in class on Wednesday 26 February 2025 (week 8), in class. Write answers on a separate piece of 8.5 by 11 inch paper, well organized and well labelled, with each solution starting on the left margin of the page. Or, print a copy of this page and write on it.

5 points per problem, total 20 points.

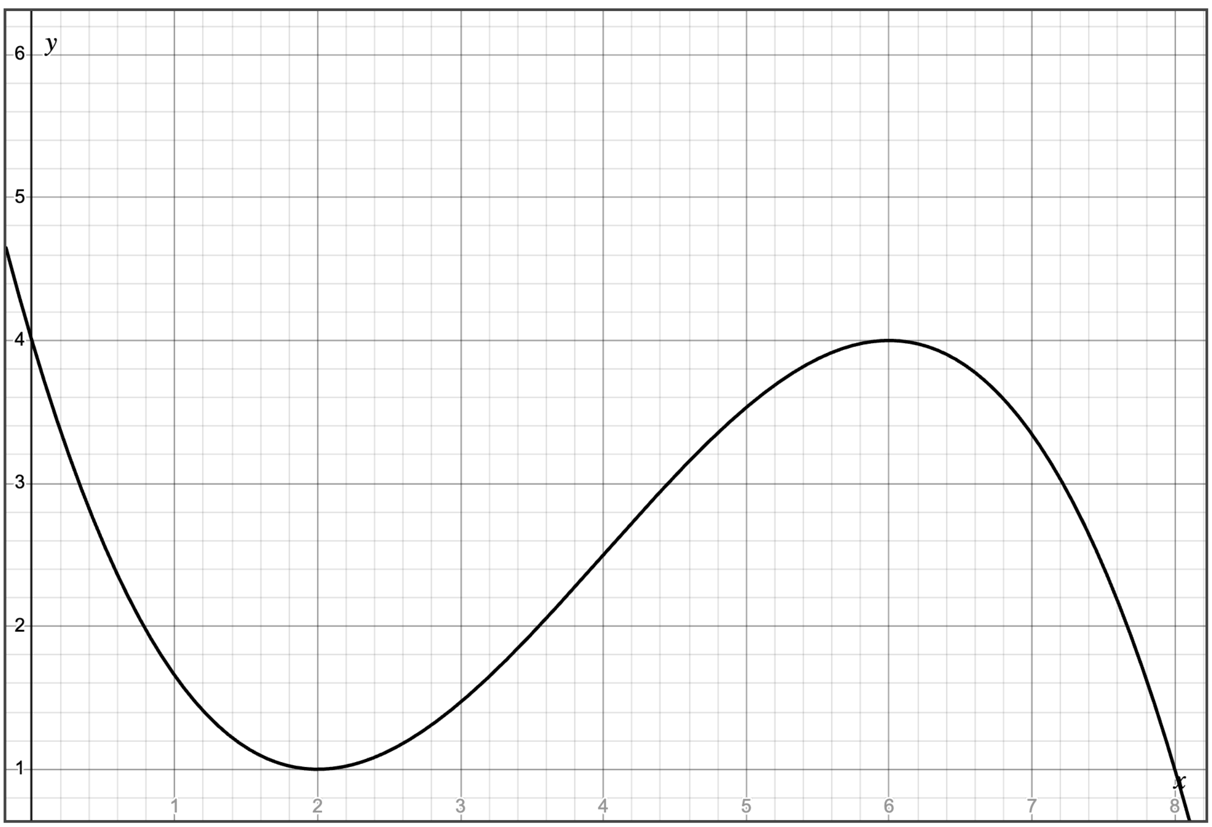
In each problem, either draw the graph of a function satisfying the stated conditions, or explain why no such function can exist. Show scales on the axes. In all parts, the function is supposed to be defined and differentiable on the interval $(0, 8)$.

1. $g'(x) \leq 0$ for all x in $(0, 8)$, $g(2) = 1$, and $g(7) = 3$.

Solution: No such function can exist. Since $g'(x) \leq 0$ for all x in $(0, 8)$, the function g must be nonincreasing ("decreasing" in the OpenStax textbook; see pages 19–20) on $(0, 8)$. Therefore $g(7) \leq g(2)$.

2. $f'(x) < 0$ on $(0, 2)$, $f'(2) = 0$, $f'(x) > 0$ on $(2, 6)$, $f'(6) = 0$, $f'(x) < 0$ on $(6, 8)$, $f(2) = 1$, and $f(6) = 4$.

Solution: Here is the graph of one possibility, out of many:



(The function used was $f(x) = 4 - \frac{3}{32}x(x-6)^2$. You were not required to give a formula for your function.)

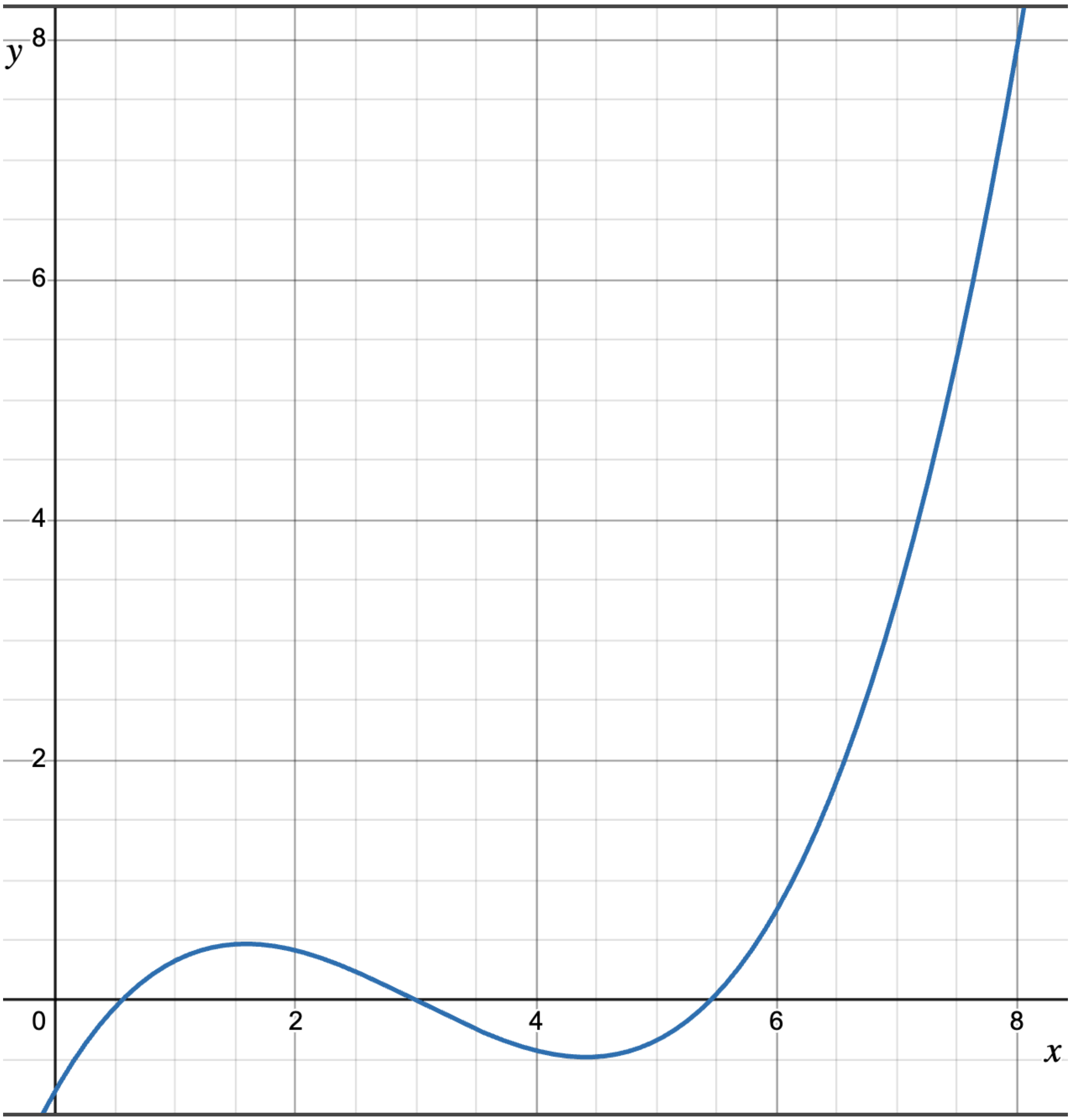
3. $k'(x) < 0$ on $(0, 4)$, $k'(4) = 0$, $k'(x) > 0$ on $(4, 8)$, and k has a local maximum at $x = 4$.

Solution: No such function can exist. Since $k'(x) < 0$ on $(0, 4)$, the function k must be strictly decreasing on $(0, 4)$. Similarly, k is strictly increasing on $(4, 8)$. Therefore k has a local minimum at $x = 4$, not a local maximum.

Date: 10 February 2025.

4. $l''(x) < 0$ on $(0, 3)$, $l''(x) > 0$ on $(3, 8)$, $l'(3) = -\frac{1}{2}$, and $l(3) = 0$.

Solution: Here is the graph of one possibility, out of many:



(The function used was $l(x) = \frac{1}{12}(x^3 - 9x^2 + 21x - 9)$. You were not required to give a formula for your function.)