Names and student IDs: ________________________________

Instructions: Turn in one copy per group, with all names and student IDs on it, by the end of class today.

You will need formulas for the areas of rectangles, triangles, and trapezoids. (The area of the trapezoid which occurs here is the difference of the areas of two triangles.)

1. Let \( f \) be the constant function \( f(x) = 7 \) for all real \( x \). Define a new function \( F_1 \) as follows: for \( t > 0 \), \( F_1(t) \) is the area of the region bounded by the \( x \)-axis, the graph of \( f \), the vertical line \( x = 0 \), and the vertical line \( x = t \).

Use your knowledge of areas of rectangles to find an explicit formula for \( F_1(t) \). Include a picture.

2. Repeat Problem 1, with the same function \( f \), but using the new function \( F_2 \) such that for \( t > -5 \), \( F_2(t) \) is the area of the region bounded by the \( x \)-axis, the graph of \( f \), the vertical line \( x = -5 \), and the vertical line \( x = t \).

3. Find \( F'_1(t) \) and \( F'_2(t) \). How do they relate to the function \( f \)?

Date: 8 January 2018.
4. Let $g$ be the function $g(x) = \frac{1}{3}x$ for all real $x$. Define a new function $G_1$ as follows: for $t > 0$, $G_1(t)$ is the area of the region bounded by the $x$-axis, the graph of $g$, the vertical line $x = 0$, and the vertical line $x = t$. (You don’t actually need the vertical line $x = 0$; it is included to emphasize the similarity with other problems.)

Use your knowledge of areas of triangles to find an explicit formula for $G_1(t)$. Include a picture.

5. Repeat Problem 4, with the same function $g$, but using the new function $G_2$ such that for $t > 5$, $G_2(t)$ is the area of the region bounded by the $x$-axis, the graph of $g$, the vertical line $x = 5$, and the vertical line $x = t$.

Use your knowledge of areas of trapezoids to find an explicit formula for $G_2(t)$. Include a picture.

6. Find $G_1'(t)$ and $G_2'(t)$. How do they relate to the function $g$?

7. Recall that if $H$ is a function, then $H'(t)$ is the instantaneous rate of change of $H$, and is also the slope in the linear approximation to $H$ at $t$. Try to imagine and explain why the derivatives of the area functions in Problems 1, 2, 4, and 5 are the original functions. Start with those in Problems 1 and 2.