This homework sheet is due in class on Friday 12 January 2018 (week 1).
All the requirements in the sheet on general instructions for homework apply. In particular, show your work (unlike WeBWorK), give exact answers (not decimal approximations), and use correct notation. (See the web page on notation.)

1. Suppose you know a function $F$ such that $F'(x) = \sin(x^2)$ for all real $x$. In terms of $F$, find all antiderivatives of the function $23 \sin(x^2)$.

In this problem, “showing your work” means showing that your supposed antiderivatives really have the right derivative.

Solution: The answer is:

$$23F(x) + C$$

for an arbitrary constant $C$.

The requested calculation (showing more steps than necessary) is:

$$\frac{d}{dx}(23F(x) + C) = 23 \frac{d}{dx}(F(x)) + \frac{d}{dx}(C) = 23F'(x) + 0 = 23\sin(x^2).$$

2. Suppose you are given a function $f$, and you know a function $F$ such that $F'(x) = f(x)$ for all real $x$. In terms of $F$, find all antiderivatives of the function $-43f(x) + 3$.

In this problem, “showing your work” means showing that your supposed antiderivatives really have the right derivative.

Solution: The answer is:

$$-43F(x) + 3x + C$$

for an arbitrary constant $C$.

The requested calculation (showing more steps than necessary) is:

$$\frac{d}{dx}(-43F(x) + 3x + C) = -43 \frac{d}{dx}(F(x)) + \frac{d}{dx}(3x) + \frac{d}{dx}(C) = -43F'(x) + 3 + 0 = -43f(x) + 3.$$

3. Suppose you are given a function $g$, and you know a function $G$ such that $G'(x) = g(x)$ for all real $x$. In terms of $G$, find all antiderivatives of the function $g(2x)$.

In this problem, “showing your work” means showing that your supposed antiderivatives really have the right derivative.

Hint: The expression $G(2x)$ should appear in your answer.

Solution: The answer is:

$$\frac{1}{2}G(2x) + C$$

for an arbitrary constant $C$.

The requested calculation, showing more steps than necessary, and using the Chain Rule at the second step (this part must be shown) is:

$$\frac{d}{dx}\left(\frac{1}{2}G(2x) + C\right) = \frac{1}{2} \frac{d}{dx}(G(2x)) + \frac{d}{dx}(C) = \frac{1}{2}(2G'(2x)) + 0 = G'(2x) = g(2x).$$

Date: 12 January 2018.
4. Suppose you know a function $G$ such that $G'(x) = \sqrt{3 + \cos(x)}$ for all real $x$. In terms of $G$, find all antiderivatives of the function $x\sqrt{3 + \cos(x^2)}$.

In this problem, “showing your work” means showing that your supposed antiderivatives really have the right derivative.

Hint: The expression $G(x^2)$ should appear in your answer.

Solution: The answer is:

$$\frac{1}{2}G(x^2) + C$$

for an arbitrary constant $C$.

The requested calculation, showing more steps than necessary, and using the Chain Rule at the second step (this part must be shown) is:

$$\frac{d}{dx} \left( \frac{1}{2}G(x^2) + C \right) = \frac{1}{2} \frac{d}{dx} (G(x^2)) + \frac{d}{dx} (C) = \frac{1}{2} (2xG'(x^2)) + 0 = xG'(x^2) = x\sqrt{3 + \cos(x^2)}.$$