The first problems are on estimation. To do the definite integrals on the rest of this worksheet, you will need the list of “recognizable derivatives”, and you will need to be prepared to adjust constants, simplify or otherwise modify integrands, and recognize a few cases in what the Chain Rule is relevant.

1. Here is the graph of a function $y = f(x)$:

What are the best numbers $r$ and $R$ you can easily find for which you can be sure the bounds

$$r \leq \int_{-2}^{10} f(x) \, dx \leq R$$

are correct? (If you are counting grid squares, you are doing more work than asked for.)

2. Here is the graph of a function $y = g(x)$:

What are the best numbers $r$ and $R$ you can easily find for which you can be sure the bounds

$$r \leq \int_{-2}^{10} g(x) \, dx \leq R$$

are correct? (If you are counting grid squares, you are doing more work than asked for.)

Date: 17 January 2018.
3. Find the following definite integrals. Give exact values. (Don’t use a calculator.) Answers must be simplified.

\[ \int_{0}^{2} x^3 \, dx = \]
\[ \int_{2}^{11} 3 \sin(x) \, dx = \]
\[ \int_{1}^{2} (x^4 - 5x^2) \, dx = \]
\[ \int_{1}^{5} \frac{x^2 - 6}{x} \, dx = \]

(Hint: Simplify the integrand.)

\[ \int_{2}^{6} \cos(2x) \, dx = \]

(Hint: The Chain Rule is relevant.)

\[ \int_{0}^{10} xe^{-x^2} \, dx = \]

(Hint: The Chain Rule is relevant.)

\[ \int_{-1}^{2} \frac{x^2}{1 + x^6} \, dx = \]

(Hint: The Chain Rule is relevant.)