1. Here are the graphs of the curves $y^2 = x$ and $y = x^3$.

We want to find the area of the region between these two graphs.

a. First, find the coordinates of the two points at which the graphs intersect. (What equations do you need to solve?)

b. Now, consider a very narrow vertical rectangle contained in this region, at $x = \frac{1}{2}$ and with width $\Delta x$. Approximately what is the area of this rectangle? (You will need to use the formulas for the functions. You should not simplify your answer.)

c. Next, consider a very narrow vertical rectangle contained in this region, at $x = \frac{1}{4}$ and with width $\Delta x$. Approximately what is the area of this rectangle? (You should not simplify your answer.)

d. Suppose now that we cover the region with many very narrow vertical rectangles, and add up their areas. Does this look like a Riemann sum for a definite integral? Which one?

e. Find the area of the region.

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2. Here are the graphs of the functions \( y = -x^2 \) and \( y = \sqrt{3 + x^4} - 3 \).

Set up an integral which gives the area between these two curves. (You will need to solve for the \( x \)-coordinates of the points at which they intersect.) Don’t try to evaluate the integral.

3. Here are the graphs of the curves \( y = e^{x/4} \), \( y = 2e^{x/4} \), \( y = 1 \), and \( y = 2 \).

a. Find the area of the region bounded by those two curves. (Using the methods from above, it will be the sum of two integrals.)

b. Now find the area of the same region by thinking of the areas of thin horizontal rectangles with height \( \Delta y \).