1. (10 points.) Consider the region between the graph of \( y = \sin(x^2) \), the \( x \)-axis, \( x = \sqrt{\pi} \), and \( x = \sqrt{2\pi} \). It is rotated about the \( y \)-axis. Find the volume of the resulting solid.

2. (5 points.) The planet Yuggxth is a sphere with radius 6000 kilometers. (It has no mountains, no valleys, and no oceans.) The density of its atmosphere at height \( h \) kilometers above the surface, measured in millions of metric tons per cubic kilometer, is \( 1.2 \exp(-0.13h) \). Write down an integral which represents the total mass of the part of the atmosphere of Yuggxth which is between the surface of the planet and 100 kilometers above the surface. Be sure to take account of the fact that the planet is not flat. Include an explanation. Don’t try to evaluate the integral.

3. (5 points.) A postmodern monument consists of a flat slab of concrete in the shape of a circle of radius 10 meters. It is to be painted with a mixture of pale green paint and pale purple paint, with the proportions of the two colors varying across the concrete, but using one liter of paint per square meter of surface. At distance \( r \) meters from the center, the concrete is painted with \( e^{-r} \) liters of pale green paint per square meter and \( 1 - e^{-r} \) liters of pale purple paint per square meter. Set up an integral which represents the total amount of pale green paint needed for this monument. Include an explanation. Don’t try to evaluate the integral.

4. (15 points.) The city of Megalopolis is circular and has a radius of 10 miles. Its population density \( r \) miles from the city center is \( \left( \frac{1}{2} \right)(30,000 - 30r^2) \) people per square mile. Find the average distance residents of Megalopolis live from the center of the city. Include an explanation.

5. (10 points.) On 1 January 1970, Wang’s Widgets Inc. was selling widgets at the rate of 200,000 widgets per year. Its rate of sales over the next three decades is modelled as \( 200,000e^{0.02t} \) widgets per year \( t \) years after 1970. According to this model, how many widgets did Wang’s Widgets Inc. sell during the period from 1 January 1970 through 31 December 1999 (a total of 30 years)? Include an explanation.

   The originally posted version of the problem had a misprint: \( 200,000e^{0.02t} \) widgets per year instead of \( 200,000e^{0.02t} \). The solution is written for the intended version. The solution for the originally posted version differs only in small details, as described afterwards.

6. (10 points.) A 5 meter chain is lying on the floor of a room with a flat floor and a 10 meter high ceiling. It has a hook at one end. Its linear density \( x \) meters from the end with the hook is \( 4 + 2\cos(\pi x) \) kilograms per meter. How much work is required to lift the chain so that it hangs straight down from the ceiling, with the hook at the ceiling?