Math 252

Sample Questions for Midterm 1

At least 80% of the points on the real exam will be modifications of problems from the problems below, homework problems (particularly written homework), worksheet problems, and problems from the sample and real Midterms 0. Note, though, that the exact form of the functions to be integrated could vary substantially, and the methods required to do them might occur in different combinations.

Be sure to get the notation right! (This is a frequent source of errors.) You have seen the correct notation in the book, in handouts, in files posted on the course website, and on the blackboard; use it. The right notation will help you get the mathematics right, and incorrect notation will lose points.

Here is the instruction sheet for Midterm 1:

(1) DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

(2) Closed book, except for a 3 × 5 file card.

(3) The following are all prohibited: Calculators (of any kind), cell phones, laptops, iPods, electronic dictionaries, and any other electronic devices or communication devices. All electronic or communication devices you have with you must be turned completely off and put inside something (pack, purse, etc.) and out of sight.

(4) The point values are as indicated in each problem; total 100 points.

(5) Write all answers on the test paper. Use the back of the page with the extra credit problems for long answers or scratch work.

(6) Show enough of your work that your method is obvious. Be sure that every statement you write is correct. Cross out any material you do not wish to have considered. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, will receive no credit.

(7) Be sure you say what you mean, and use correct notation. Credit will be based on what you say, not what you mean.

(8) When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(2)$, or $\frac{2\pi}{9}$. Decimal approximations will not be accepted.

(9) Final answers must always be simplified unless otherwise specified.

(10) Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).

(11) Time: 50 minutes.
Problems.

1. (6 points.) Calculate the Riemann sum to approximate $\int_1^9 2x \, dx$ using 4 equal length subintervals and right endpoints.

2. (6 points.) Write down, but do not evaluate, the Riemann sum to approximate $\int_{-1}^2 \sin\left(\frac{1}{2}x\right) \, dx$ using 6 equal length subintervals and left endpoints.

3. (10 points.) Write down, but do not evaluate, the Riemann sum to approximate $\int_2^6 \cos(2x) \, dx$ using $n$ equal length subintervals and left endpoints. (Your answer will use the symbol $\sum$.)

4. (10 points.) Find the average value of the function $k(t) = te^{-t^2}$ on the interval $[1, 3]$.

5. (9 points.) The population of fire-breathing monsters on the planet Yuggxth $t$ years after the beginning of the year 4000, given in millions of fire-breathing monsters, is modelled by $P(t) = 40(1 - e^{-2t})$.

According to the model, what is average population of fire-breathing monsters during the period from the beginning of the year 4007 to from the beginning of the year 4011?

6. (10 points.) A particle moves along a straight line in such a way that, $t$ hours after it starts moving, its distance to the right of its starting point is $\sin(t) + 2t$ feet. During the period from 2 hours to 5 hours after its starting time, which is the average distance of the particle to the right of its starting point?

7. (10 points.) Let $a$ be a constant. Find $\int te^{-at} \, dt$.

8. (10 points.) Find $\int_1^e \frac{(u - 1)^2}{u} \, du$.

9. (10 points.) Find $\int x\sqrt{1-x} \, dx$.

10. (10 points.) Let $H$ be a function such that $H'(t) = 2\sin(5t^2)$ for all real $t$. Find $\int tH(t) \, dt$ in terms of elementary functions and $H$. 
11. (10 points.) Define \( F(x) = \int_1^x \frac{\sin(t)}{t^2} \, dt \) for all real \( x \). Find \( \int_2^6 \frac{\sin(2x)}{x^2} \, dx \) in terms of elementary functions and \( F \).

12. (10 points.) Let \( H \) be a function such that \( H'(t) = \cos(5t^3) \) for all real \( t \). Find \( \int 10t^3 \cos(5t^{12}) \, dt \) in terms of elementary functions and \( H \).

13. (8 points.) Your friend’s mother claims that
\[
\int \frac{1}{(1 + x^2)^2} \, dx = \frac{1}{2} \left( \frac{x}{1 + x^2} + \arctan(x) \right) + C.
\]
Because she has previously made mistakes, you are skeptical of this answer. Check whether this answer is right. Show your work.

14. (10 points.) Define \( h(s) = \int_{-8}^{2s^3} \frac{\arctan(t)}{\sqrt{e^t + t^2}} \, dt \). Find \( h'(s) \).

15. (10 points.) Define \( s(x) = x^3 \int_{-3}^x \cos(e^t) \, dt \). Find \( s'(x) \).

16. (10 points.) Define
\[
g(x) = \begin{cases} 
-x^2 + 2x + 1 & x \leq 2 \\
x - 1 & x > 2.
\end{cases}
\]
Find \( \int_1^4 g(x) \, dx \).

17. (10 points.) Find \( \int y^2 \sin(2y) \, dy \).

18. (10 points.) Find \( \int e^y \sin(2y) \, dy \).

19. (6 points.) Let \( g \) be a continuous function defined on \([-2, 9]\), and suppose that
\[
\int_{-1}^2 g(x) \, dx = 5, \quad \int_{-1}^1 g(x) \, dx = 3, \quad \int_2^4 g(x) \, dx = 12, \quad \text{and} \quad \int_4^8 g(x) \, dx = 4.
\]
Find \( \int_1^8 g(x) \, dx \). (You may not need to use all the information given.)
20. (6 points.) Let \( g \) be a continuous function defined on \([-2, 9]\), and suppose that
\[
\int_{-1}^{2} g(x) \, dx = 5, \quad \int_{-1}^{1} g(x) \, dx = 3, \quad \int_{2}^{4} g(x) \, dx = 12, \quad \text{and} \quad \int_{4}^{8} g(x) \, dx = 4.
\]
Find \( \int_{8}^{2} g(x) \, dx \). (You may not need to use all the information given.)

21. (6 points.) Let \( g \) be a continuous function defined on \([-2, 9]\), and suppose that
\[
\int_{-1}^{2} g(x) \, dx = 5, \quad \int_{-1}^{1} g(x) \, dx = 3, \quad \int_{2}^{4} g(x) \, dx = 12, \quad \text{and} \quad \int_{4}^{8} g(x) \, dx = 4.
\]
Find \( \int_{1}^{2} 6g(x) \, dx \). (You may not need to use all the information given.)

22. (6 points.) Find \( \int_{1}^{4} \frac{\sin(\sqrt{z})}{\sqrt{z}} \, dz \).

23. (10 points.) Find \( \int \frac{\sin(z) \cos(z)}{1 + \sin^2(z)} \, dz \).

24. (7 points.) Here is the graph of a function \( y = f(x) \):

Find \( \int_{1}^{6} f(x) \, dx \).
25. (6 points.) Here is the graph of a function $y = f(x)$:

![Graph of $y = f(x)$]

Find $\int_{0}^{2} 3f(x) \, dx$.

26. (10 points.) Find an antiderivative $F$ of the function $f(x) = \sqrt{x} + x + 7$ such that $F(4) = 30$. 