Math 252 Final Exam
NAME:________________________
Th 12 Dec. 1996
Student ID:____________________

GENERAL INSTRUCTIONS

1. DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

2. The exam pages are **two sided**.

3. Closed book, except for a graphing calculator and a 3 × 5 file card. Note: you may not use the calculator as a substitute for calculus. (Note: The exams this quarter will not permit calculators of any kind.)

4. The following are all prohibited: Cell phones, laptops, iPods, electronic dictionaries, and any other electronic devices or communication devices. All electronic or communication devices you have with you must be turned completely off and put inside something (pack, purse, etc.) and out of sight.

5. The point values are as indicated in each problem; total 200 points.

6. Write all answers on the test paper. Extra paper is provided at the end for long answers or scratch work.

7. Show enough of your work that your method is obvious. Be sure that every statement you write is correct. Cross out any material you do not wish to have considered. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, will receive no credit.

8. Be sure you say what you mean, and use correct notation. Credit will be based on what you say, not what you mean.

9. When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(2)$, or $\frac{2\pi}{9}$. Calculator approximations will not be accepted.

10. Final answers must always be simplified unless otherwise specified.

11. Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).

12. Time: 120 minutes.

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1. (10 points/part) A flare is fired into the air at time $t = 0$ seconds from the top of a cliff 60 feet above the ocean below. The upward velocity of the flare at time $t$ seconds is given by the function $v(t)$ which is graphed below.

(a) Did the flare go up or down between times $t = 2$ and $t = 6$? How much?

(b) When was the flare highest? Justify your answer.

(c) The light from the flare went out at exactly 10 seconds. How high was the flare at that time?

2. (5 points) If $f(x)$ is measured in kilograms/meter$^3$ and $x$ is measured in meters, what are the units of $\int_a^b f(x) \, dx$?
3. (10 points/part) Use the techniques of integration you have learned to find the following integrals. Be sure to show your steps. The function \( G(x) \) is defined by

\[
G(x) = \int_0^x \sqrt{1 + t^3} \, dt
\]

for \( t \geq -1 \). The answers to the parts of this problem should be expressed in terms of the usual elementary functions and (if needed) \( G(x) \).

Note that there are 5 parts to this problem, two of them on the next page.

(a) \[ \int_3^7 \sqrt{1 + 8x^3} \, dx \]

(b) \[ \int_0^\infty z e^{-3z} \, dz \]

(Show that the answer to (b) is exactly \( \frac{1}{9} \).)

(c) \[ \int w G(w) \, dw \]
Problem 3 (continued); see instructions at the top of the previous page.

(d) \[ \int x \cos(ax^2) \, dx \]

(a is a constant.)

(e) \[ \int_{0}^{1} \frac{1}{2e^t} \, dt \]

(Show that the answer to (e) is exactly \( \frac{1}{2} - \frac{1}{2e} \).)

4. (10 points) Find

\[ \frac{d}{dx} \left( \int_{-1}^{x} \sin \left(1 - \frac{1}{2}t^3\right) \, dt \right). \]

There should be no integral signs in your answer.
5. (20 points) A 727 jet needs to be moving 200 miles/hour to take off. If it can accelerate from 0 to 200 miles/hour in 40 seconds, how long must the runway be? (Assume constant acceleration.)

6. (15 points) The region between the curve $y = \arctan(x)$, the $x$-axis, and the line $x = 17$, is rotated about the $x$-axis. Write an integral which expresses the volume of the resulting solid of revolution. Do not evaluate the integral.
7. (10 points/part) Test the following integrals for convergence or divergence. You need not evaluate them.

(a) \[ \int_0^\infty \frac{1}{1 + \frac{1}{2} x^4} \, dx \]

(b) \[ \int_0^1 \frac{1}{1 + \sqrt{t}} \, dt \]

(c) \[ \int_0^\infty \frac{e^{-x}}{2x} \, dx \]
8. (20 points) The kinetic energy of a particle of mass \( m \) moving at speed \( v \) is \( \frac{1}{2}mv^2 \). Set up an integral which gives the kinetic energy of a phonograph record of mass 60 grams and radius 10 centimeters rotating at \( 200/3 \) revolutions per minute. Do not evaluate the integral.

9. (20 points) At the end of 1996, the annual consumption of iron in Lamantina was projected to grow linearly, from a value of 10,000,000 pesos/year at the beginning of 1997 to a value of 20,000,000 pesos/year at the beginning of 2017. (The peso is the unit of currency of Lamantina.) Assuming an interest rate of 7%, write an integral which expresses the present value of the total projected iron consumption in Lamantina for the next 20 year period from the beginning of 1997 to the beginning of 2017. Do not evaluate the integral.
Extra credit. (This problem will only be counted if you get a grade of B or better on the main part of this exam.)

Consider the solid of revolution obtained by rotating the region between the $x$-axis and the curve $y = e^{-x^2}$ about the $y$-axis.

(a) Calculate the volume of this solid by considering cylindrical shells centered on the $y$-axis (or by some other method which gives a closed form answer).

(b) Calculate the volume of this solid in terms of $\int_{-\infty}^{\infty} e^{-x^2} \, dx$ by considering vertical slices perpendicular to the $x$-axis.

(c) Combine your answers to parts (a) and (b) to show that $\int_{-\infty}^{\infty} e^{-x^2} \, dx = \sqrt{\pi}$. 