1. (10 points.) Define
\[ g(z) = \int_{3}^{e^z} \frac{\sin(t)}{t^6 + 3} \, dt. \]
Find \( g'(z) \).

2. (14 points.) Let \( G \) be a function such that \( G'(x) = 9x \sec^2(x^3) \) for all real \( x \). Find
\[ \int G(x) \, dx \]
in terms of elementary functions and \( G \).

3. (15 points.) Solve the initial value problem
\[ y'(x) = xe^{x-y(x)}, \quad y(0) = \ln(3). \]

4. (12 points.) Let \( a > 0 \) be a constant. Find the average value of the function
\[ f(x) = \frac{8e^x}{\sqrt{1 + ae^x}} \]
on the interval \([\ln(2), \ln(3)]\).

5. (15 points.) A chemical reaction in a solution involves combining two molecules of compound A to produce a molecule of compound B. The reaction occurs when two molecules of compound A collide, so the concentration \( a(t) \) of compound A at time \( t \) is governed by the differential equation
\[ a'(t) = -ka(t)^2 \]
for some constant \( k > 0 \).

Suppose that \( t \) is measured in minutes, that \( a(t) \) is measured in moles per liter, that when the reaction begins the concentration of compound A is \( \frac{1}{2} \) mole per liter, and that 2 minutes later the concentration of compound A is \( \frac{1}{8} \) mole per liter. Find \( k \).

6. (15 points.) Find an antiderivative \( F \) of the function
\[ f(x) = \frac{12}{(x-2)(x+1)} \]
such that \( F(3) = 4 \).

7. (12 points.) A water tank on the seventh moon of the planet Glorppxth has the shape of an inverted cone, with height 4 feet and radius 3 at the top. It is full, and you want to empty it by pumping all the water over the rim. The seventh moon of Glorppxth is quite small, and water there weighs only 2 pounds per cubic foot. How much work is required?

8. (8 points.) Determine whether or not the improper integral
\[ \int_{0}^{1} \frac{7 + 4 \sin(6x)}{x} \, dx \]
converges.

Date: 17 March 2021.
9. (8 points.) A circular plate with radius 3 centimeters has a variable density, namely the density at a point which is \( r \) centimeters from the center is \( 9 - r^2 \) grams per square centimeter. Find the total mass of the plate.

10. (12 points.) Consider the region between the curve \( x = y^{-4} \), the \( y \)-axis, and the line \( y = 2 \). It is rotated about the \( x \)-axis. Find its volume (possibly infinite).

11. (12 points.) Find the area of the region bounded by the curves \( y = \frac{1}{4}x^2 \), \( y = -x \), and \( y = 2x - 3 \), and with \( 0 \leq x \leq 2 \). (It may help to draw a picture of the region first.)

12. (8 points.) You want to find \( \int (1 + 4t^2)^{7/2} \, dt \). Using a suitable substitution, convert this indefinite integral into one involving trigonometric functions but only integer powers.

13. (8 points.) A chemical reaction in a solution involves combining a molecule of compound A and a molecule of compound B to produce a molecule of compound C. The reaction rate is proportional to the product of the concentrations of compound A and compound B. (This is a constant times the rate at which molecules of compound A collide with molecules of compound B.) Suppose that the initial concentration of compound A is \( a_0 \) the initial concentration of compound B is \( b_0 \), and compound C is not present.
   
   Set up a differential equation which models the amount of compound C in the solution as a function of time. Be sure to state what your variables mean. You will have an unknown constant in your equation; state that it is an unknown constant, and state whether it is positive or negative.

14. (8 points.) Determine whether or not the function \( y(t) = 7e^{2t} - 2t^4 - 2t - 1 \) satisfies the differential equation \( y'(t) = 2y(t) + 4t^2 \).