FINAL EXAM SAMPLE PROBLEMS

The final exam is comprehensive. It will probably have greater emphasis on material covered since Midterm 2. Formally there may appear to be fewer problems on earlier material, since earlier material is often used in problems on later material.

There is only one attempt at the final exam, unlike for the midterms.

Most problems will be similar to `WeBWorK` problems, written homework problems, and the problems here. Note, though, that the exact form of functions which appear could vary substantially.

Be sure to get the notation right! (This is a frequent source of errors.) The right notation will help you get the mathematics right, and I will complain about incorrect notation.

The problems here have point values attached, which give a very rough idea of the point values problems requiring a similar amount of work will have on the real exam. Note, however, that the real midterm will total 200 points, and the problems here total somewhat more than that.

The final exam from the last time I taught the course should give a reasonable idea of the length to expect.

Be sure to get the notation right! (This is a frequent source of errors.) You have seen the correct notation in the book, in handouts, in files posted on the course website, and on the blackboard; *use it*. The right notation will help you get the mathematics right, and incorrect notation will lose points.

Instructions for the final exam via Zoom are on the course home page, linked at:

`https://pages.uoregon.edu/ncp/Courses/Math252_W21_Web/FinalExam/FinalExam.html`.

(This link should be clickable.) The main difference from those for Midterm 2 is the length and time. Remember that work must be written in an organized and logical manner, and must say in correct notation what you did. This includes correct use of parentheses, writing equals signs when you are saying two things are equal, and much more; see:

`https://pages.uoregon.edu/ncp/Courses/Math252_W21_Web/CourseInfo/GenHWInstr.pdf`

and

`https://pages.uoregon.edu/ncp/Courses/Math252_W21_Web/CourseInfo/Notation/Notation.html`

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(These links should be clickable.)

Here are the mathematical instructions for the final exam:

(1) The point values are as indicated in each problem; total 200 points (not counting extra credit).

Date: 12 March 2021.

(2) Show enough of your work that your method is obvious. Be sure that every statement you write is correct. Cross out any material you do not wish to have considered. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, will receive no credit.
(3) Be sure you say what you mean, and use correct notation. Credit will be based on what you say, not what you mean.
(4) When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(2)$, or $\frac{2\pi}{9}$. Calculator approximations will not be accepted.
(5) Final answers must always be simplified unless otherwise specified.

Extra credit will be given to the first two people catching any error in the sample problems or their solutions, provided I am notified before corrections are posted. You must specify both the error and the correct version.

1. Problems on new material

This section consists of sample problems on material covered since Midterm 2.

1. (8 points.) Determine whether or not the function $y(t) = 6e^{2t} - e^t$ satisfies the differential equation $y'(t) = 2y(t) + e^t$. Show your work.

2. (8 points.) Determine whether or not the function $y(t) = 3e^{2t} + e^t$ satisfies the differential equation $y'(t) = 2y(t) + e^t$. Show your work.

3. (15 points.) A chemical reaction in a solution involves combining three molecules of compound A to produce a molecule of compound X. The reaction occurs when three molecules of compound A all collide at the same time, so the concentration $a(t)$ of compound A at time t is governed by the differential equation

$$a'(t) = -ka(t)^3$$

for some constant $k > 0$.

Suppose that t is measured in hours, that $a(t)$ is measured in moles per liter, that when the reaction begins the concentration of compound A is 1 mole per liter, and that 2 hours later the concentration of compound A is $\frac{1}{2}$ moles per liter. Find k.

4. (18 points.) Find all solutions to the differential equation

$$y'(x) = \frac{2y(x)^2}{1 + x^2}.$$

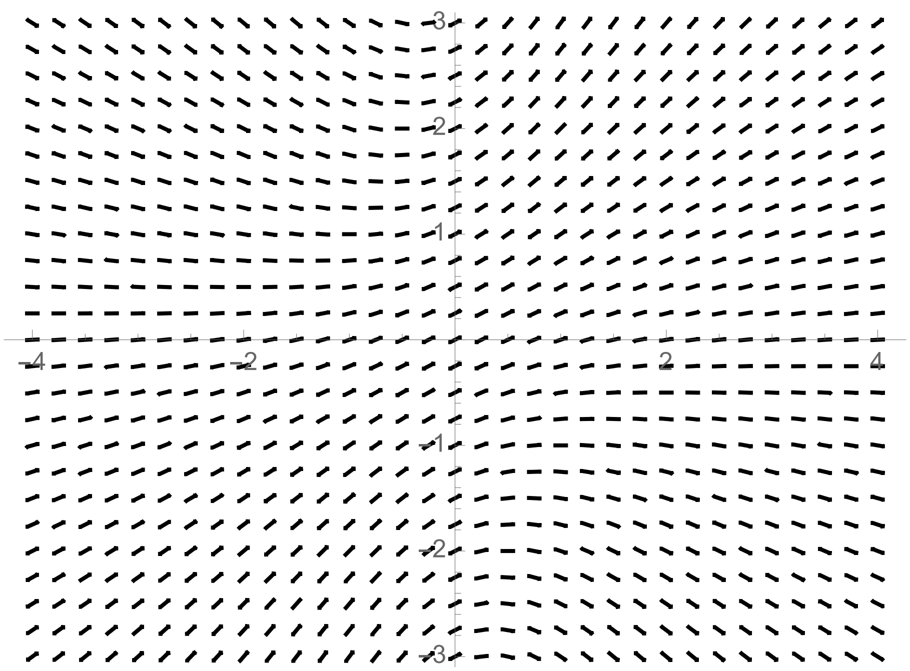
Find the solution $y(x)$ satisfying $y(0) = \frac{1}{6}$, and for this solution determine $\lim_{x\to\infty} y(x)$.

5. (12 points.) Find all solutions to the differential equation $y'(x) = \cos(x)[2y(x) - 4]^2$.

6. (8 points.) In the absence of other effects, the population of lemmings on an island grows at a rate proportional to the existing population. In addition, there is a wolf on this island, which eats 100 lemmings per year. (The wolf can't reproduce, since there is only one.) Moreover, 20 lemmings per year swim to a neighboring island to escape from the wolf.

Set up a differential equation which models the population of lemmings on this island as a function of time. Be sure to state what your variables mean. You will have one unknown constant in your equation; state that it is unknown constant, and state whether it is positive or negative.

7. (10 points.) Here is a direction field:



On this graph, draw the graphs of the five solutions $y(x)$ to the equations satisfying the initial conditions $y(0) = -2, -1, 0, 1, 2$.

8. (10 points.) A vat in a campus bar starts out at the beginning of finals week with 200 liters of orange juice. Vodka is added to it at 40 liters per day, the contents are kept well mixed, and 40 liters per day of the mixture are sold to students who have finished their final exams. (So the students finishing later get more vodka.)

Set up, but do not solve, a differential equation which models the amount of vodka in the vat as a function of time. Be sure to state what your variables mean.

9. (15 points.) Consider the differential equation

$$y'(x) = kxe^{-y(x)},$$

in which k is a constant. Suppose a solution $x \mapsto y(x)$ to this equation satisfies $y(0) = 0$ and $y(2) = \ln(5)$. Find k.

10. (10 points.) Consider the differential equation

$$f'(x) = \frac{\sin(x)}{7 + \sin(f(x))}.$$

Give an equation which implicitly determines the general solution to this equation.

11. (12 points.) Solve the initial value problem

$$y'(x) = \frac{x}{y(x)}, \qquad y(1) = 2.$$

12. (10 points.) At its creation, a dungeons and dragons universe contains 100 goblins. The population grows at a rate proportional to the existing population. If the population of goblins doubles after 6 years, how long does it take until there are 500 goblins?

2. Problems on old material

This section contains a random selection of sample problems on material covered before Midterm 2. It is **not** necessarily expected to be a good match for the problems on old material on the real final exam. (It consists of the problems from the final exam the last time I taught this course which were on material from before Midterm 2.)

1. (13 points.) Find an antiderivative F of the function $f(x) = x^2 e^{x^3} - \sec^2(x) - 12x^2$ such that $F(0) = 2$.

2. (13 points.) Find $\displaystyle\int_0^3 12t\cos(2t)\,dt$.

3. (10 points.) Consider the region between the curve $y = x^3 + x + 1$, the lines $x = 1$ and $x = 4$, and the x-axis. It is rotated about the y-axis. Set up an integral which gives the volume of the resulting solid. Do not evaluate the integral.

4. (16 points.) Find $\displaystyle\int \frac{6}{u(2-u)}\,du$.

5. (12 points.) Define

$$h(x) = \int_6^{x^2} t\cos(t^5)\,dt.$$

Find $h'(x)$.

6. (12 points.) A particle moves along a straight line in such a way that, t minutes after it starts moving, its distance to the right of its starting point is $3\sin(t) - 2t$ millimeters. During

the period from 2 minutes to 5 minutes after its starting time, what is the average distance of the particle to the right of its starting point?

7. (12 points.) A small valley on the planet Yuggxth has a parabolic cross section: measured in meters, it looks like the region above the curve $y = x^2$. A vertical dam for a water reservoir is built in this valley. It is 9 meters high. When the reservoir is full, set up (but do not evaluate) an integral which gives the force it exerts on the dam. The density of water is 1000 kilograms per cubic meter, and the gravitational constant at the surface of Yuggxth is exactly 8 meters/second^2.

8. (12 points.) A 60 pound pail of water is attached to a rope, and is lifted from a depth of 100 feet in a well. The rope weighs $\frac{1}{10}$ pounds per foot. How much work is required to lift the pail and the rope?

9. (15 points.) Determine whether or not the improper integral $\displaystyle\int_7^\infty \frac{13 + 3\sin(6e^y)}{y^5}\,dy$ converges.

10. (15 points.) Find the area of the region bounded by the curves $y = 2x^2$, $y = 3 - x$, and the x-axis, with $x \geq 0$. (It may help to draw a picture of the region first.)

11. (15 points.) A solid has a base which is the disk in the xy plane whose boundary is the circle $x^2 + y^2 = 9$. Its cross sections perpendicular to the the x-axis are squares. Find the volume of this solid.

12. (10 points.) You want to find $\displaystyle\int (t^2 - 16)^{-5/2}\,dt$, for $t > 4$. Using a suitable substitution, convert this indefinite integral into one involving trigonometric functions but only integer powers. Simplify the resulting integrand, but do not attempt to find the resulting integral.