MATH 251 (PHILLIPS) MIDTERM 0 VERSION 1

NAME: SOLUTIONS

INSTRUCTIONS: No books, notes, or calculators are permitted on this test. All answers must be simplified as much as possible. Write all answers in the spaces provided at the right. Do scratchwork on the back or on blank paper provided for this purpose. No partial credit. Time: 20 minutes.

1. Write as a single fraction, and simplify as much as possible: \( \frac{2}{p} - \frac{2}{p^2} - \frac{1}{p^3} + 5 \)

Solution:

\[
\frac{2}{p} - \frac{2}{p^2} - \frac{1}{p^3} + 5 = \frac{2(p + 5) - (p - 2)}{(p - 2)(p + 5)} = \frac{2p + 10 - p + 2}{(p - 2)(p + 5)} = \frac{p + 12}{(p - 2)(p + 5)}.
\]

You can multiply out the denominator, getting

\[
\frac{p + 12}{p^2 + 3p - 10},
\]

but that isn’t any simpler in this context, and is not necessary.

2. Let \( f(x) = 5 - x \). Evaluate the expression \( f(x + 3) - f(3x) \), and simplify it as much as possible.

Solution:

\[
f(x + 3) - f(3x) = 5 - (x + 3) - (5 - 3x) = 5 - x - 3 - 5 + 3x = 2x - 3.
\]

3. Find all real solutions to the equation \( z + 4 = 5z^{-1} \). If no real solution exists, write “no solution”.

Solution:

\[
z + 4 = 5z^{-1}
\]

Multiply by \( z \):

\[
z^2 + 4z = 5
\]

\[
z^2 + 4z - 5 = 0
\]

\[
(z - 1)(z + 5) = 0
\]

\[
z = 1 \quad \text{or} \quad z = -5.
\]

Note that both answers actually are solutions to the original equation, that is, that multiplying both sides by \( z \) at the first step did not introduce any extraneous solutions.

Since there is no partial credit, no credit is given for only one of the two solutions.

4. Simplify the following expression as much as possible. If no simplification is possible, write “not possible”:

\[
\frac{2t^2 + 2t}{2t^2 + 8t}
\]

Solution:

\[
\frac{2t^2 + 2t}{2t^2 + 8t} = \frac{2t(t + 1)}{2t(t + 4)} = \frac{t + 1}{t + 4}.
\]

The last expression can’t be further simplified.
5. Find all real numbers $a$ such that $(-a, 3)$ is in the first quadrant (and not on any of the coordinate axes).

Solution: $(-a, 3)$ is in the first quadrant if and only if $-a > 0$, which happens if and only if $a < 0$.

6. Find all real solutions to the equation $4\left(\frac{1}{x^2} + 3\right) = 12$. If no real solution exists, write “no solution”.

Solution:

\[
\frac{1}{x^2} + 3 = 3
\]

\[
\frac{1}{x^2} = 0
\]

Multiply both sides by $x^2$ to get $1 = 0$. Therefore there are no solutions. (Alternatively, it is obvious that $\frac{1}{x^2}$ can never be zero.)

7. Multiply out: $(x - 2)(x^2 - x - 7)$.

Solution:

\[
(x - 2)(x^2 - x - 7) = x^3 - 2x^2 - x^2 + 2x - 7x + 14 = x^3 - 3x^2 - 5x + 14.
\]

8. Let $f(x) = mx^4 - \tan(x) \ln(x)$, for a fixed constant $m$. Find $f'(x)$.

Solution: We use the product rule on the second term, remembering to distribute the minus sign correctly. Thus

\[
f'(x) = m \cdot 4x^3 - \left[ \sec^2(x) \ln(x) + \tan(x) \cdot \frac{1}{x} \right] = 4mx^3 - \sec^2(x) \ln(x) - \tan(x) \cdot \frac{1}{x}.
\]

9. Let $q(s) = (ks^4 - \sin(s))^9$, for a fixed constant $k$. Find $q'(s)$.

Solution: We use the chain rule, getting

\[
q'(s) = 9(ks^4 - \sin(s))^8 \cdot \frac{d}{ds} (ks^4 - \sin(s))
\]

\[
= 9(ks^4 - \sin(s))^8 (k \cdot 4s^3 - \cos(s)) = 9(3ks^4 - \sin(s))^8 (34ks^3 - \cos(s)).
\]

10. For the graph of the function $y = f(x)$ sketched here, determine which of the numbers $-4, -1, 0, 1, 4$ is closest to $f'(-1)$, or that $f'(-1)$ does not exist.
Solution: The graph at the right is the same, except with the addition of the tangent line at $x = -1$. The slope is positive and much greater than 1, so the correct answer is 4. (In fact, $f'(1)$ is exactly 4.)