Submit with your exam the statement (presumably handwritten), with your signature, “I understand the instructions on academic integrity for this exam, and will follow them. I understand that being found responsible for academic misconduct is considered cheating and will result in a score of zero on this exam.”

GENERAL INSTRUCTIONS (The problems are on the next page.)
(1) The point values are as indicated; total 100 points (not counting extra credit).
(2) This exam is open book and open notes. All work must be your own; you may not collaborate with others. This means that no interactive help (in person, internet based, or any other method) is allowed. A graphing calculator (not on your computer) is allowed, but computer algebra systems are not. All electronic devices other than your computer, cell phone (if needed for scanning), and calculator must be off and out of reach; the cell phone must be visible but off when not being used for scanning. Normal rules on academic integrity apply.
(3) Starting time 29 January 2021 at 8:00 am Pacific Standard Time. Your work must be submitted by 9:05 am Pacific Standard Time. For a nominally 50 minute exam, this allows 5 minutes for downloading at the beginning and 10 minute for scanning etc. at the end. You must stay until 9:00 am, even if you are done. Use the time to check answers, or read a book.
(4) Show enough of your work that your method is obvious. Be sure that every statement you write is correct and uses correct notation. Cross out any material you do not wish to have considered. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, will receive no credit.
(5) Be sure you say what you mean, and use correct notation. Credit will be based on what you say, not what you mean.
(6) When exact values are specified, give answers such as \( \frac{1}{7} \), \( \sqrt{2} \), \( \ln(2) \), or \( \frac{2\pi}{9} \). Calculator approximations will not be accepted.
(7) Final answers must always be simplified unless otherwise specified.

EXCERPTS FROM THE INSTRUCTIONS FOR REMOTE TESTING (See the separate sheet for the full set of instructions.)
(1) You must be present in the normal class Zoom meeting (ID 916 3410 5223) while taking the exam. Zoom settings:
   • Virtual background off.
   • Video and web camera on, and set to show you and your workspace.
   • Microphone on.
   • Speaker on (to hear announcements).
(2) Submit questions to the instructor via chat. They will be answered via chat, unless they are something the whole class should know about, in which case they will be answered via a spoken announcement.
(3) Alternate submission, in case you have trouble with Canvas: by email to ncp@uoregon.edu. The file must be contained in the email message; files stored on Google Docs or similar sites will not be retrieved. (I will check the time stamps on email.)

Date: 29 January 2021.
1. (1 point) Are you awake?

Solution: I hope you were!

2. (10 points.) Here is the graph of a function $y = h(x)$:

![Graph of $y = h(x)$](image)

Find $\int_{2}^{8} h(x) \, dx$.

Solution: We interpret $\int_{1}^{6} h(x) \, dx$ as the area between the graph and the $x$-axis, taking areas below the $x$-axis to be negative. Thus, using the formula for the area of a triangle for the first two terms,

$$
\int_{2}^{8} h(x) \, dx = \int_{2}^{4} h(x) \, dx + \int_{4}^{5} h(x) \, dx + \int_{5}^{8} h(x) \, dx = \frac{1}{2}(2)(2) - \frac{1}{2}(1)(1) - (1)(3) = 2 - \frac{1}{2} - 3 = -\frac{3}{2}.
$$

(One can also just count grid boxes and halves of grid boxes.)

Remember that “mixed fractions” are incorrect notation in mathematics courses: $-\frac{1}{2}$ is evaluated as the product $-1 \cdot \frac{1}{2} = -\frac{1}{2}$, which is the wrong answer.

3. (10 points.) Calculate and simplify the Riemann sum to approximate $\int_{-5}^{1} (2x + 7) \, dx$ using 3 equal length subintervals and left endpoints.

Solution: Set $f(x) = 2x + 7$. The intervals have length $(1 - (-5))/3 = 2$, so they are $[-5, -3]$, $[-3, -1]$, and $[-1, 1]$. Therefore the Riemann sum is

$$f(-5) \cdot 2 + f(-3) \cdot 2 + f(-1) \cdot 2 = (-3) \cdot 2 + 1 \cdot 2 + 5 \cdot 2 = 6.$$
It is wrong to write $\int_{-5}^{1} (2x+7) \, dx = 6$. This statement is false. The integral $\int_{-5}^{1} (2x+7) \, dx$ is equal to 18, not 6. One might write $\int_{-5}^{1} (2x+7) \, dx \approx 6$, but, because the subintervals are so long, this is a very poor approximation.

4. (12 points.) Define

$$S(x) = \begin{cases} 2x+1 & x \leq 3 \\ \sin(x) & x > 3. \end{cases}$$

Find $\int_{-2}^{7} S(x) \, dx$.

**Solution:** We can write

$$\int_{-2}^{7} S(x) \, dx = \int_{-2}^{3} S(x) \, dx + \int_{3}^{7} S(x) \, dx = \int_{-2}^{3} (2x+1) \, dx + \int_{3}^{7} \sin(x) \, dx.$$  

We have

$$\int_{-2}^{3} (2x+1) \, dx = (x^2 + x) \bigg|_{-2}^{3} = 3^2 + 3 - (-2)^2 - (-2) = 9 + 3 - 4 + 2 = 10$$

and

$$\int_{3}^{7} \sin(x) \, dx = -\cos(x) \bigg|_{3}^{7} = \cos(7) - \cos(3).$$

Therefore

$$\int_{-2}^{7} S(x) \, dx = 10 + \cos(7) - \cos(3).$$

5. (10 points.) Find an antiderivative $F$ of the function $f(x) = 3\sqrt{x}+x+7$ such that $F(4) = 30$.

**Solution:** We first need

$$\int (3\sqrt{x}+x+7) \, dx = \int (3\sqrt{x}+x+7) \, dx = 3 \left( \frac{2}{3} x^{3/2} + \frac{x^2}{2} + 7x + C = 2x^{3/2} + \frac{x^2}{2} + 7x + C. \right.$$

Now we must choose $C$ such that the function $F(x) = 2x^{3/2} + \frac{1}{2} x^2 + 7x + C$ satisfies $F(4) = 30$. Thus,

$$30 = F(4) = 2 \cdot 4^{3/2} + \frac{1}{2} \cdot 4^2 + 7 \cdot 4 + C = 2 \cdot 8 + 8 + 28 + C = 52 + C,$$

so $C = -22$. Thus, we take

$$F(x) = 2x^{3/2} + \frac{x^2}{2} + 7x - 22.$$
6. (12 points.) Define 
\[ g(z) = \int_{-11}^{2z-1} \frac{\sin(t)}{8-t} \, dt \]
for \( t \) in \([-11, 4]\). Find \( g'(z) \).

*Solution:* Define 
\[ F(x) = \int_{-11}^{x} \frac{\sin(t)}{8-t} \, dt. \]
Then (using the Fundamental Theorem of Calculus for the second formula)
\[ g(z) = F(2z-1) \quad \text{and} \quad F'(x) = \frac{\sin(x)}{8-x}. \]
Using the chain rule, we get
\[ g'(z) = \frac{d}{dz} F(2z-1) = F'(2z-1) \frac{d}{dz} (2z-1) = \frac{\sin(2z-1)}{8-(2z-1)} \cdot 2 = \frac{2\sin(2z-1)}{9-2z}. \]
The simplification of the denominator is required.
(The variable \( t \) may not appear anywhere in the answer, because \( g'(z) \) must be a function of \( z \).)

7. (12 points.) Find \( \int_{0}^{\pi} \frac{\cos(x)}{7+3\sin(x)} \, dx \).

*Solution:* Use the substitution \( u = 7 + 3\sin(x) \), so \( du = 3\cos(x) \, dx \) and \( \cos(x) \, dx = \frac{1}{3} \, du \), to get (using \( 7 + 3\sin(x) > 0 \) at the last step)
\[ \int \frac{\cos(x)}{7+3\sin(x)} \, dx = \int \left( \frac{1}{3} \right) \left( \frac{1}{u} \right) \, du = \left( \frac{1}{3} \right) \ln(|u|) + C \]
\[ = \frac{1}{3} \ln(|7 + 3\sin(x)|) + C = \frac{1}{3} \ln(7 + 3\sin(x)) + C. \]
So
\[ \int_{0}^{\pi} \frac{\cos(x)}{7+3\sin(x)} \, dx = \left. \frac{1}{3} \ln(7 + 3\sin(x)) \right|_{0}^{\pi} \]
\[ = \frac{1}{3} \ln(7 + 3\sin(\pi)) - \frac{1}{3} \ln(7 + 3\sin(0)) = \frac{1}{3} \ln(7) - \frac{1}{3} \ln(7) = 0. \]
The simplification is required.

8. (12 points.) Let \( H \) be a function such that \( H'(x) = e^{4x^3} \) for all real \( x \). Find 
\[ \int xH(x) \, dx \]
in terms of elementary functions and \( H \).
**Solution:** Integrate by parts, taking
\[ u(x) = H(x), \quad v'(x) = x, \quad u'(x) = e^{4x^3}, \quad \text{and} \quad v(x) = \frac{1}{2}x^2. \]
Thus,
\[ \int xH(x)\,dx = \frac{1}{2}x^2H(x) - \int \frac{1}{2}x^2e^{4x^3}\,dx. \]
We do the remaining integral using the substitution \( w = 4x^3 \). Thus, \( dw = 12x^2\,dx \), so \( x^2\,dx = \frac{1}{12}\,dw \), and
\[ \int \frac{1}{2}x^2e^{4x^3}\,dx = \int \left( \frac{1}{2} \right) \left( \frac{1}{12} \right) e^w\,dw = \frac{1}{24}e^w + C_0 = \frac{1}{24}e^{4x^3} + C_0. \]
So (with \( C = -C_0 \))
\[ \int xH(x)\,dx = \frac{1}{2}x^2H(x) - \int \frac{1}{2}x^2e^{4x^3}\,dx = \frac{1}{2}x^2H(x) - \frac{1}{24}e^{4x^3} - C_0 = \frac{1}{2}x^2H(x) - \frac{1}{24}e^{4x^3} + C. \]
Just writing the last line shows enough work.

9. (10 points) Let \( a \) be a constant. Find \( \int e^{2ax}\,dx \).

**Solution:** We use the substitution \( u = 2ax \), so \( du = 2a\,dx \), and \( dx = \frac{1}{2a}\,du \). Thus, since \( a \) is a constant,
\[ \int e^{2ax}\,dx = \int \left( \frac{1}{2a} \right) e^u\,du = \frac{1}{2a} \int e^u\,du = \left( \frac{1}{2a} \right) e^u + C = \frac{e^{2ax}}{2a} + C. \]

10. (11 points) Let \( f \) be a function defined on \(( -\infty, \infty) \), and assume that \( f, f', \) and \( f'' \) are all continuous. Suppose that
\[ f(3) = 10, \quad f'(3) = -1, \quad \text{and} \quad f''(3) = 12, \]
that
\[ f(6) = -30, \quad f'(6) = 100, \quad \text{and} \quad f''(6) = 80, \]
and that
\[ \int_0^3 f(x)\,dx = 31 \quad \text{and} \quad \int_3^6 f(x)\,dx = -22. \]
Find the exact value of \( \int_3^6 xf''(x)\,dx \).

**Solution:** We integrate by parts, taking \( u = x, \quad v'(x) = f''(x), \quad u'(x) = 1, \) and \( v(x) = f'(x) \). This gives (using the Fundamental Theorem of Calculus at the second step):
\[ \int_3^6 xf''(x)\,dx = xf'(x)\bigg|^6_3 - \int_3^6 f'(x)\,dx = xf'(x)|^6_3 - f(x)|^6_3 \]
\[ = 6f'(6) - 3f'(3) - [f(6) - f(3)] = 6 \cdot 100 - 3(-1) - (-30) + 10 = 643. \]
Extra credit. (15 extra credit points.) (Do not attempt this problem until you have finished all the ordinary problems on the exam and checked your answers to them. It problem will only be counted if you get at least 75 points on the main part of this exam.)

When the brakes of a car are applied hard, the result is a fixed constant deceleration until the car comes to a stop. Use antiderivatives to show that if the car is going twice as fast when the brakes are applied, it travels four times as far before it stops.

Solution: Let $a$ be the constant deceleration (chosen positive), and let $v_0$ be the initial velocity the first time. Let $f(t)$ be the position $t$ units of time after applying the brakes at velocity $v_0$, and let $g(t)$ be the same thing for initial velocity $2v_0$.

Antidifferentiating twice gives $f(t) = -\frac{at^2}{2} + c_1 t + c_2$ for constants $c_1$ and $c_2$. Since $f(0) = 0$, we get $c_2 = 0$. Since $f'(0) = v_0$, we get $c_1 = v_0$. So $f(t) = -\frac{at^2}{2} + v_0 t$. The car stops when $f'(t) = 0$, and solving the equation shows this happens at $t = \frac{v_0}{a}$. The distance travelled is then $f(\frac{v_0}{a}) = \frac{v_0^2}{2a}$.

Calculating $g$ the same way, we get $g(t) = -\frac{at^2}{2} + 2v_0 t$, $g'(t) = 0$ when $t = \frac{2v_0}{a}$ (the time to stop is twice as long), and $g(\frac{2v_0}{a}) = 2\frac{v_0^2}{a}$, which is four times as far.

You can also notice that the distance is proportional to $v_0^2$ in the result for $f$, so doubling $v_0$ quadruples the distance.