MIDTERM 2 SAMPLE PROBLEMS

Midterm 2 will cover through the end of Section 6.6 of the book. Most problems will be similar to WebWork problems, written homework problems, and the problems here. Note, though, that the exact form of functions which appear could vary substantially.

Be sure to get the notation right! (This is a frequent source of errors.) The right notation will help you get the mathematics right, and I will complain about incorrect notation.

The problems here have point values attached, which give a very rough idea of the point values problems requiring a similar amount of work will have on the real exam. Note, however, that the real midterm will total 100 points, and the problems here total much more than that.

Midterm 2 from the last time I taught the course should give a reasonable idea of the length to expect.

Be sure to get the notation right! (This is a frequent source of errors.) You have seen the correct notation in the book, in handouts, in files posted on the course website, and on the blackboard; use it. The right notation will help you get the mathematics right, and incorrect notation will lose points.

Instructions for Midterm 2 via Zoom are on the course home page, at: https://pages.uoregon.edu/ncp/Courses/Math252_W21_Web/Midterm2/ExamInstructions_M2/ExamInstructions_M2.pdf. (Copy and paste the link; clicking on it doesn’t work, because of the line breaks.) The main difference from those for Midterm 1 is that they are more explicit about showing work in a coherent manner. Work must be written in an organized and logical manner, and must say in correct notation what you did. This includes correct use of parentheses, writing equals signs when you are saying two things are equal, and much more; see: https://pages.uoregon.edu/ncp/Courses/Math252_W21_Web/CourseInfo/GenHWInstr.pdf and https://pages.uoregon.edu/ncp/Courses/Math252_W21_Web/CourseInfo/Notation/Notation.html.

(These links should be clickable.)

Here are the mathematical instructions for Midterm 2:

(1) The point values are as indicated in each problem; total 100 points (not counting extra credit).

(2) Show enough of your work that your method is obvious. Be sure that every statement you write is correct. Cross out any material you do not wish to have considered. Correct answers with insufficient justification or accompanied by additional incorrect statements.

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will not receive full credit. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, will receive no credit.

(3) Be sure you say what you mean, and use correct notation. Credit will be based on what you say, not what you mean.

(4) When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(2)$, or $\frac{2\pi}{9}$. Calculator approximations will not be accepted.

(5) Final answers must always be simplified unless otherwise specified.

Extra credit will be given to the first two people catching any error in the sample problems or their solutions, provided I am notified before corrections are posted. You must specify both the error and the correct version.

Problems.

1. (10 points.) Evaluate the definite integral $\int_{1}^{\infty} \frac{\ln(y)}{y} \, dy$, or else show that it does not converge.

2. (10 points.) Evaluate the definite integral $\int_{0}^{\infty} \frac{e^{3y}}{4 + e^{6y}} \, dy$, or else show that it does not converge.

3. (12 points.) Evaluate the definite integral $\int_{-1}^{1} \frac{e^{x}}{e^{x} - 1} \, dx$, or else show that it does not converge.

4. (8 points.) Determine whether or not the improper integral $\int_{5}^{\infty} \frac{2 + 7 \arctan(12t)}{t} \, dt$ converges.

5. (8 points.) Determine whether or not the improper integral $\int_{1}^{\infty} (8 - 3 \cos(11y)) e^{-y} \, dy$ converges.

6. (10 points.) Let $H$ be a function such that $H'(t) = \cos(5t^3)$ for all real $t$. Find

$$\int 10t^3 \cos(5t^{12}) \, dt$$

in terms of elementary functions and $H$. 
7. (10 points.) Define 

\[ h(t) = \arctan(t) \int_6^t s \sin^2(e^s) \, ds. \]

Find \( h'(s) \).

8. (8 points.) Determine whether or not the improper integral \( \int_0^2 \frac{9 + 2 \sin(1/x)}{x^2} \, dx \) converges.

9. (12 points.) Find the area of the region bounded by the curves \( y^2 = x + 1 \), and \( y = (x + 1)^3 \). (It may help to draw a picture of the region first.)

10. (15 points.) Find the area of the region bounded by the curves \( y = -\frac{1}{2}x^2 \), \( y = x \), and \( y = 3 - 2x \), and with \( 0 \leq x \leq 2 \). (It may help to draw a picture of the region first.)

11. (9 points.) Write down a formula in terms of integrals for the area of the region bounded by the curves \( y = x^2 \), \( y = (x - 4)^2 \), and \( y = (x - 2)^2 - 4 \). (It may help to draw a picture of the region first.) Do not evaluate the integrals. You need not do any simplification.

12. (9 points.) Let \( G \) be a function such that \( G'(r) = r^2 \sin(3r^4) \) for all real \( r \). Find 

\[ \int G(r) \, dr \]

in terms of elementary functions and \( G \).

13. (8 points.) A postmodern monument consists of a flat slab of concrete in the shape of a circle of radius 8 meters. It is to be painted with a mixture of pale orange paint and pale blue paint, with the proportions of the two colors varying across the concrete, but using two liters of paint per square meter of surface. At distance \( y \) meters north of the center, the concrete is painted with \( 2e^{-y^2} \) liters of pale orange paint per square meter and \( 2 - 2e^{-y^2} \) liters of pale blue paint per square meter. Set up an integral which represents the total amount of pale orange paint needed for this monument. Include an explanation. Don’t try to evaluate the integral.

14. (10 points.) A lake is shaped like an upside down hemisphere, with radius 100 meters. The temperature of the water at depth \( z \) meters below the surface is \( 20 - \frac{z}{10} \) degrees Centigrade. Set up an integral which represents the average temperature of the water in this lake. Include an explanation. Don’t try to evaluate the integral.

15. (15 points.) Find the centroid (center of mass) of the region between the curve \( y = 4 - x^2 \) and the \( x \)-axis.
16. (15 points.) Find the $x$-coordinate of the centroid (center of mass) of the region between the curve $y = e^{-x}$, the $y$-axis, and the $x$-axis. (The region extends infinitely far to the right.)

17. (8 points.) Consider the region between the curve $y = e^{-x}$, the lines $x = 1$ and $x = 4$, and the $x$-axis. It is rotated about the line $y = -2$. Write down a formula in terms of integrals for the volume of the resulting solid. Do not evaluate the integrals. You need not do any simplification.

18. (15 points.) Consider the region between the curve $y = e^x + x + 1$, the lines $x = 0$ and $x = 2$, and the $x$-axis. It is rotated about the $y$-axis. Find the volume of the resulting solid.

19. (9 points.) Let $H$ be a function such that $H'(x) = e^{5x^3}$ for all real $x$. Find

$$\int xH(x) \, dx$$

in terms of elementary functions and $H$.

20. (12 points.) Consider the region between the curves $y = e^{-x}$ and $y = e^{-2x}$, and the lines $x = 1$ and $x = \ln(6)$. It is rotated about the $x$-axis. Find the volume of the resulting solid.

21. (15 points.) A spiky pyramid-like solid has square horizontal cross sections. The total height is 2 meters, and for $0 \leq h \leq 2$ the cross section at height $h$ meters has side length $(2 - h)^2$ meters. Find the volume of this solid.

22. (9 points.) Let $h$ be a continuous function defined for all real numbers, and let $H$ be a function such that $H'(x) = h(x)$ for all real $x$. Find $\int_2^3 x^2h(x^3) \, dx$. Your answer may involve $h$, $H$, or both.

23. (15 points.) Consider the region below the graph of $y = e^{-x^2}$ and in the first quadrant. It is rotated about the $y$-axis. Find the volume (possibly infinite) of the resulting solid.

24. (10 points.) Let $a$ be a constant. Find $\int \cos^2(at) \, dt$.

25. (12 points.) Find $\int \cos^2(5x) \sin^3(5x) \, dx$.

26. (15 points.) Find $\int \tan^5(3z) \, dz$. 
27. (12 points.) Find \( \int \frac{x}{(x + 1)(2x + 1)} \, dx \).

28. (12 points.) Find \( \int \frac{1}{(4 + x^2)^{3/2}} \, dx \).

29. (12 points.) Find \( \int x^2 \sec^4(2x^3) \tan^2(2x^3) \, dx \).

30. (12 points.) Find \( \int \frac{1}{(1 - 4x^2)^{3/2}} \, dx \).

31. (8 points.) A nonlinear industrial size spring has restoring force \( 2x + \frac{1}{3}x^3 \) Newtons when stretched \( x \) meters beyond its natural (equilibrium) length. Find the work done when stretching it from 1 meter beyond its natural length to 2 meters beyond its natural length.

32. (12 points.) Find \( \int \frac{2t^2 + t}{1 + t^2} \, dt \).

33. (10 points.) A cable on the fourth moon of the planet Groggxth is 2 meters long and has linear density \( \frac{3}{1 + x^2} \) kilograms per meter at the point \( x \) meters from the heavy end. It is hanging from a hook in the basement of a building, with the heavy end at the top. Set up an integral which represents the amount of work needed to pull up the cable and lay it flat on the floor of the main level. The floor is one meter higher than the hook on which the cable hangs, and the acceleration due to gravity on the surface of the fourth moon of Groggxth is 4 m/sec\(^2\). Do not attempt to evaluate the integral.

34. (10 points.) A cable on the planet Kzorqb is 2 meters long and has linear density \( x \) kilograms per meter at the point \( x \) meters from the heavy end. It is lying on the ground. The amount of work needed to hang the cable from a hook 4 meters above the ground with the light end at the top is 16 Newton-meters. What is the acceleration due to gravity on the surface of Kzorqb?

35. (12 points.) You are building a stone pyramid on the planet Yuggxth. The stone you use is flat on the ground, and has a density of 5000 kilograms per cubic meter. The pyramid has a square base, which is 20 meters on each side, and is 10 meters high; its sides are straight. The acceleration due to gravity on the surface of Yuggxth is 10 m/sec\(^2\). How much work is needed to construct the pyramid?

36. (10 points.) A particle moves along a straight line in such a way that, \( t \) hours after it starts moving, its distance to the right of its starting point is \( 3t^2 + 6t \) centimeters. During the period
37. (15 points.) A lake on the planet Yuggxth is held in place by a dam which is shaped like an upside down semicircle with radius 7 meters. The lake is filled with a poisonous liquid (which, however, is very nutritious to the fire breathing monsters which live there) with a density of 500 kilograms per cubic meter. The acceleration due to gravity on the surface of Yuggxth is \( 4 \text{ m/sec}^2 \). What force does the lake exert on the dam?

38. (9 points.) Define

\[
g(z) = \int_{-1}^{\sin(z)} \frac{\sin(t)}{t^4 + 1} dt.
\]

Find \( g'(z) \).

39. (9 points.) Let \( q \) be a continuous function defined for all real numbers, and let \( Q \) be a function such that \( Q'(x) = q(x) \) for all real \( x \). Find

\[
\int \frac{g(\arctan(x))}{1 + x^2} dx.
\]

Your answer may involve \( g, G, \) or both.