1. (13 points.) Find an antiderivative $F$ of the function $f(x) = x^2e^x - \sec^2(x) - 12x^2$ such that $F(0) = 2$.

2. (13 points.) Find $\int_0^3 12t \cos(2t)\,dt$.

3. (10 points.) Here is a direction field:

On this graph, draw the graphs of the five solutions $y(x)$ to the equations satisfying the initial conditions $y(0) = -2, -1, 0, 1, 2$.

4. (10 points.) Consider the region between the curve $y = x^3 + x + 1$, the lines $x = 1$ and $x = 4$, and the $x$-axis. It is rotated about the $y$-axis. Set up an integral which gives the volume of the resulting solid. Do not evaluate the integral.

5. (10 points.) A vat in a campus bar starts out at the beginning of finals week with 200 liters of orange juice. Vodka is added to it at 40 liters per day, the contents are kept well mixed, and 40 liters per day of the mixture are sold to students who have finished their final exams. (So the students finishing later get more vodka.)

Set up, but do not solve, a differential equation which models the amount of vodka in the vat as a function of time. Be sure to state what your variables mean.

6. (16 points.) Find $\int \frac{6}{u(2-u)}\,du$. 


7. (12 points.) Define

\[ h(x) = \int_0^{x^2} t \cos(t^5) \, dt. \]

Find \( h'(x) \).

8. (15 points.) Consider the differential equation

\[ y'(x) = kxe^{-y(x)}, \]

in which \( k \) is a constant. Suppose a solution \( x \mapsto y(x) \) to this equation satisfies \( y(0) = 0 \) and \( y(2) = \ln(5) \). Find \( k \).

9. (12 points.) A particle moves along a straight line in such a way that, \( t \) minutes after it starts moving, its distance to the right of its starting point is \( 3 \sin(t) - 2t \) millimeters. During the period from 2 minutes to 5 minutes after its starting time, what is the average distance of the particle to the right of its starting point?

10. (12 points.) A small valley on the planet Yuggxth has a parabolic cross section: measured in meters, it looks like the region above the curve \( y = x^2 \). A vertical dam for a water reservoir is built in this valley. It is 9 meters high. When the reservoir is full, set up (but do not evaluate) an integral which gives the force it exerts on the dam. The density of water is 1000 kilograms per cubic meter, and the gravitational constant at the surface of Yuggxth is exactly 8 meters/second\(^2\).

11. (12 points.) A 60 pound pail of water is attached to a rope, and is lifted from a depth of 100 feet in a well. The rope weighs \( \frac{1}{10} \) pounds per foot. How much work is required to lift the pail and the rope?

12. (15 points.) Determine whether or not the improper integral

\[ \int_7^\infty \frac{13 + 3 \sin(6e^y)}{y^5} \, dy \]

converges.

13. (15 points.) Find the area of the region bounded by the curves \( y = 2x^2 \), \( y = 3 - x \), and the \( x \)-axis, with \( x \geq 0 \). (It may help to draw a picture of the region first.)

14. (15 points.) A solid has a base which is the disk in the \( xy \) plane whose boundary is the circle \( x^2 + y^2 = 9 \). Its cross sections perpendicular to the \( x \)-axis are squares. Find the volume of this solid.

15. (10 points.) At its creation, a dungeons and dragons universe contains 100 goblins. The population grows at a rate proportional to the existing population. If the population of goblins doubles after 6 years, how long does it take until there are 500 goblins?

16. (10 points.) You want to find \( \int (t^2 - 16)^{-5/2} \, dt \), for \( t > 4 \). Using a suitable substitution, convert this indefinite integral into one involving trigonometric functions but only integer powers. Simplify the resulting integrand, but do not attempt to find the resulting integral.

EC1. (15 extra credit points.) A circular hole is drilled straight through the center of a solid sphere. The hole is exactly 6 inches long. What is the volume of the part of the sphere which remains after the hole has been drilled? Be sure to simplify your answer completely.
EC2. (10 extra credit points.) Finish the integral in Problem 17. There should be no parts in your answer of the form trigonometric function evaluated at inverse trigonometric function of something.