MATH 252 (PHILLIPS): SOLUTIONS TO WRITTEN HOMEWORK 1.

This homework is due on Canvas on Friday 8 January 2020 (week 1), at 10:00 pm.
All the requirements in the sheet on general instructions for written homework apply.

1. (6 points.) Suppose you know a function $F$ such that $F'(x) = \sin(x^2)$ for all real $x$.
In terms of $F$, find all antiderivatives of the function $23\sin(x^2)$.

In this problem, “showing your work” means showing that your supposed antiderivatives really have the right derivative.

**Solution:** The answer is: $23F(x) + C$

for an arbitrary constant $C$.

The requested calculation (showing more steps than necessary) is:

$$\frac{d}{dx}(23F(x) + C) = 23\frac{d}{dx}(F(x)) + \frac{d}{dx}(C) = 23F'(x) + 0 = 23\sin(x^2).$$

2. (6 points.) Suppose you are given a function $f$, and you know a function $F$ such that $F'(x) = f(x)$ for all real $x$.
In terms of $F$, find all antiderivatives of the function $-43f(x) + 3$.

In this problem, “showing your work” means showing that your supposed antiderivatives really have the right derivative.

**Solution:** The answer is: $-43F(x) + 3 + C$

for an arbitrary constant $C$.

The requested calculation (showing more steps than necessary) is:

$$\frac{d}{dx}(-43F(x) + 3 + C) = -43\frac{d}{dx}(F(x)) + \frac{d}{dx}(3x) + \frac{d}{dx}(C) = -43F'(x) + 3 + 0 = -43f(x) + 3.$$

3. (6 points.) Suppose you are given a function $g$, and you know a function $G$ such that $G'(x) = g(x)$ for all real $x$.
In terms of $G$, find all antiderivatives of the function $g(2x)$.

In this problem, “showing your work” means showing that your supposed antiderivatives really have the right derivative.

Hint: The expression $G(2x)$ should appear in your answer.

**Solution:** The answer is: $\frac{1}{2}G(2x) + C$

for an arbitrary constant $C$.

The requested calculation, showing more steps than necessary, and using the Chain Rule at the second step (this part must be shown) is:

$$\frac{d}{dx}\left(\frac{1}{2}G(2x) + C\right) = \frac{1}{2}\frac{d}{dx}(G(2x)) + \frac{d}{dx}(C) = \frac{1}{2}(2G'(2x)) + 0 = G'(2x) = g(2x).$$

4. (6 points.) Suppose you know a function $G$ such that $G'(x) = \sqrt{3 + \cos(x)}$ for all real $x$.
In terms of $G$, find all antiderivatives of the function $x\sqrt{3 + \cos(x^2)}$.

_**Date:** 8 January 2021._
In this problem, “showing your work” means showing that your supposed antiderivatives really have the right derivative.

Hint: The expression $G(x^2)$ should appear in your answer.

Solution: The answer is: 

$$
\frac{1}{2}G(x^2) + C
$$

for an arbitrary constant $C$.

The requested calculation, showing more steps than necessary, and using the Chain Rule at the second step (this part must be shown) is:

$$
\frac{d}{dx} \left( \frac{1}{2}G(x^2) + C \right) = \frac{1}{2} \frac{d}{dx}(G(x^2)) + \frac{d}{dx}(C) = \frac{1}{2}(2xG'(x^2)) + 0 = xG'(x^2) = x\sqrt{3 + \cos(x^2)}.
$$