The base of each rectangle will be one of the subintervals in the partition.
The height will be function value at some point in that subinterval.
Right hand Riemann sum: use the right endpoints, so $t_1 = x_1$, $t_2 = x_2$, $t_3 = x_3$.
The tops of the rectangles are the green lines.
The tops of the rectangles for the left sum are the purple lines. (less easy to see).
Area of first green rectangle: height $f(x_1)$, base $x_1 - x_0$. Area: $f(x_1)(x_1 - x_0)$.
Area of second green rectangle: height $f(x_2)$, base $x_2 - x_1$. Area: $f(x_2)(x_2 - x_1)$.
Area of third green rectangle: height $f(x_3)$, base $x_3 - x_2$. Area: $f(x_3)(x_3 - x_2)$.
The Riemann sum is $f(x_1)(x_1 - x_0) + f(x_2)(x_2 - x_1) + f(x_3)(x_3 - x_2)$.
The left Riemann sum is gotten using the left endpoints.
It is $f(x_0)(x_1 - x_0) + f(x_1)(x_2 - x_1) + f(x_2)(x_3 - x_2)$.
For red rectangles $f(t_1)(x_1 - x_0) + f(t_2)(x_2 - x_1) + f(t_3)(x_3 - x_2)$.

If we equal length subintervals, the base lengths $x_i - x_{i-1}$ are all the same.
If $f(x) = 1 + \sin(x)$ on $[-\pi, 3\pi]$, and we use left endpoints and 5 equal length subintervals, we get: all base lengths are

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$$x_0 + 2\pi, x_1 + 2\left(\frac{2\pi}{5}\right), \ldots, x_5$$

The Riemann sum is:

$$f(0)\left(\frac{2\pi}{5}\right) + f\left(\pi + \frac{2\pi}{5}\right)\frac{2\pi}{5} + f\left(\pi + \frac{4\pi}{5}\right)\frac{2\pi}{5} + f\left(\pi + \frac{6\pi}{5}\right)\frac{2\pi}{5} + f\left(\pi + \frac{8\pi}{5}\right)\frac{2\pi}{5}$$

$$= \left[1 + \sin(0)\left(\frac{2\pi}{5}\right) + 1 + \sin\left(\pi + \frac{2\pi}{5}\right)\left(\frac{2\pi}{5}\right) + \ldots + \right.$$
If $f$ is continuous on $[a,b]$ and we take the left Riemann sum with $n$ equal length subintervals, and let $n \to \infty$, (sort of like binomial expansion) the limit will be a limit, which we write

$$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x$$

infinitesimally small.

$\int$ is an "elongated S", for sum. $\Delta x$ is supposed to be a "very small $dx$".

In $\int_{a}^{b} (1 + \sin(x)) \, dx$,

most have to put parentheses with right hand sum, the same thing happens and you get the same limit.

In general, if you get the same limit using arbitrary subdivisions (but with length of intervals $\to 0$) and arbitrary sums (like red rectangles above - prov page), we say the function $f$ is integrable on $[a,b]$, and write $\int_{a}^{b} f(x) \, dx$.

The function $f(x) = \begin{cases} 1 & x \text{ rational} \\ 0 & x \text{ irrational} \end{cases}$ is not integrable (and pf).

Recall the Riemann sum

$$\left[ 1 + \sin \left( \frac{2\pi}{5} \right) \right] \left( \frac{2\pi}{5} \right) + \left[ 1 + \sin \left( \frac{4\pi}{5} \right) \right] \left( \frac{2\pi}{5} \right) + \cdots + \left[ 1 + \sin \left( \frac{8\pi}{5} \right) \right] \left( \frac{2\pi}{5} \right)$$

Shorthand: $\sum_{k=1}^{5} \left[ 1 + \sin \left( \frac{k\pi}{5} \right) \right] \left( \frac{2\pi}{5} \right)$

Ess not immediately from integrability:

$$1 + 4 + 9 + 16 + \cdots + 49 = \sum_{k=1}^{7} k^2 \quad \text{or} \quad \sum_{k=1}^{7} k^2 \quad \text{or} \quad \sum_{k=1}^{7} \frac{1}{k^2}$$

where $k=1$ $k=2$ $k=7$

Write out $\sum_{k=1}^{4} \frac{1}{k^2} \quad = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}$

when it ends

when it starts.

Summation variable.

$$\sum_{k=0}^{25} \sin(k) = \sin(0) + \sin(1) + \cdots + \sin(25)$$