We didn't exactly get a Riemann sum. It was only off by "higher order terms = dx".

We get Terms:

\[ 2\pi x (34 - x^5 - x) \Delta x + \pi (\Delta x)^3 \frac{d}{dx} (34 - x^5 - x) \]

-- volume of the shell.

\[ \text{circumference} \times \text{height} \times \text{thickness} \text{ of a cylindrical shell} \]

Linear Approximation:

If linearize this in \( \Delta x \) the variable (x is fixed), linearization at \( \Delta x = 0 \) we get only the first term: \( 2\pi x (34 - x^5 - x) \Delta x \).

When set up integrals for areas, volumes, etc., we use linear approximation. The second term gives formally \( \int_0^b \pi (34 - x^5 - x) \Delta x^2 \). All such things are zero.

Original form: difference of volumes of cylinders: \( \pi (x + \Delta x)^5 (34 - x^5 - x) - \pi x^5 (34 - x^5 - x) \).

After manipulation, gave red boxed expression.

**Example (disguised of earlier work). Area under \( y = 2x \) for \( x = 0 \) to \( 2 \).**

One of these rectangles has height \( 2x \) and width \( \Delta x \), so area

\[ 2x \Delta x \].

True area under this part of the curve is shaded in green.

The rectangle misses the red part, which has area \( (\frac{1}{2} \Delta x)(2 \Delta x) = 2\Delta x^2 \).

True area is \( 2x \Delta x + 2\Delta x^2 \). Linearization gives \( 2x \Delta x \), and error is equal to \( \int_0^2 2x \Delta x \) (using the linearization).

**Example. Consider region between \( x = \sin(y) + 1 \), \( y = \frac{\pi}{2} \) and \( y \)-axis. Roll the disk about line \( y = -3 \). Find volume.**

I will start with \( y = \sin(x) + x \).

\[ \sqrt{1 + x^2} \]

\[ \text{Slope at 0 is } 2, \text{ at } \frac{\pi}{2} \text{ is } 1. \]

For visual aid, switch x and y:

For visual aid, switch x and y:
Suppose we wished to integrate with respect to $x$. At a given value of $x$, before thickening, a vertical slice has outer radius $3 + \frac{3}{2} y$ and inner radius $3 + y$ (distance from 3 to $y$).

We would need to solve $x = \sin(y) y$ for $y$ in terms of $x$.

Need a function of $x$ because we need to integrate:

$$\int_{\frac{\pi}{2}}^{\pi} \sin(y) y \, dy$$

The cylindrical shell plus some stray minds

Volume is up to higher order terms in $\Delta x$.

$$\text{(Circumference)(Length)(Thickness)}$$

$$= 2\pi (y + 3) (\sin(y) + y) \Delta y$$

Here can easily rewrite.

Get linearized version:

$$= 2\pi (y + 3) \sin(y + y) \Delta y$$

Volume will be:

$$\int_{0}^{\pi/2} 2\pi (y + 3) \sin(y) y \, dy$$

Still need to do the integral.

(Need to multiply out the integrand.

and use one of the resulting terms by parts.)

**Ex.** Region between 0 and $\pi$, above $x$-axis, below $y = \sin(x^2)$ rotated about $y$-axis.

If use washers:

Solve $y = \sin(x^2)$ for $x$, getting two solutions in [0, $\pi$].

One is $x = \sqrt{\text{arcsin}(y)}$.

Use shells with vertical axis

Cylindrical shell

<table>
<thead>
<tr>
<th>Thickness: $\Delta x$</th>
<th>Volume (linearized) is $2\pi x \sin(x^2)$ $\Delta x$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height: $y = \sin(x^2)$</td>
<td>Volume of while they are: $\int_{0}^{\pi} 2\pi x \sin(x^2) , dx$ (Use subst.)</td>
</tr>
</tbody>
</table>

*Diagram picture:*

outer shell

inner shell

$\Gamma$