Consider region between $y = e^{-x}$, $x=1$, $x=6$, and the x-axis. Suppose units are cm, and it has uniform density $3 \text{ g/cm}^2$. Find moments about $x$ and $y$ axes, and the coordinates of the center of mass.

(Contribute): (For centroid, density will cancel out.)

\[ y = e^{-x} \]

\[ \text{picture is not good, } \frac{1}{2} \text{ is actually less than } \frac{1}{2} \]

Divide region into pieces:

Consider the pieces of shape, region between $x$ and $x+\Delta x$. ($\Delta x$ in picture is for too large)

Center of mass is at green dot (center of the rectangle).

First approximation, use a rectangle with base $\int_{x}^{x+\Delta x}$ and height $e^{-x}$ (effectively a left Riemann sum.) The x and y coordinate of center of mass is $x + \frac{1}{2} \Delta x$.

The y coord = $\frac{1}{2} e^{-x}$. If used right endpoint, would get $\frac{1}{2} e^{-(x+\Delta x)}$.

The mass is $(e^{-x}\Delta x)(3) = 3e^{-x}\Delta x$. Moments about x and y axes are:

<table>
<thead>
<tr>
<th>Height</th>
<th>Base</th>
<th>Density</th>
<th>for ( y )-axis</th>
<th>( (x+\frac{1}{2}\Delta x) ) ( 3e^{-x}\Delta x )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Distance to ( y )-axis</td>
</tr>
</tbody>
</table>

\[ \frac{3}{2} e^{-x} (3e^{-x}\Delta x) \]

\[ \text{for } x \text{-axis: } \frac{3}{2} e^{-x} (3e^{-x}\Delta x) \]

Note: Moments are given as if all mass were concentrated at the center of mass.

We discard higher order terms on $\Delta x$. ($\Delta x$ is supposed to be much smaller than shown; see green example). Green section around term contributes $\pm \Delta x \cdot 3 e^{-x} \Delta x = \frac{3}{2} e^{-x}(\Delta x)^2$. Drop it.

May as well take center of mass to have x=0 and y=0 instead of $x+\frac{1}{2} \Delta x$, since $\Delta x$ is very small.

The moment of an extended object is the sum of the moments of the pieces. Sum for total mass.

For moment about y-axis we get

\[ \sum_{i=1}^{n} x_i \cdot 3e^{-x}\Delta x; \text{corresponding integral is } \int_{1}^{6} x \cdot 3e^{-x}dx \]

Moment about x-axis is similarly, \[ \int_{1}^{6} (\frac{1}{2} e^{-x})(3e^{-x})dx \] and total mass is \[ \int_{1}^{6} 3e^{-x}dx \].

We can rewrite it: \[ 3 \int_{1}^{6} e^{-x}dx \]

The x- and y-coordinates of center of mass are

\[ \frac{3}{2} \int_{1}^{6} e^{-x}dx \]

\[ \frac{3}{2} \int_{1}^{6} e^{-x}dx \]

\[ \text{Redundancy (3 g/cm}^2 \text{) cancels out, but nothing else does.} \]

Do this using integration by parts. Do this with substitution $u = -x$.
Sec 7.1 Differential equations. The solution to a differential equation is a whole function, not just a number.

Ex. A basic exponential growth population model. Let \( P(t) \) be the population of trolls in Norway at time \( t \), \( t \) in centuries after 2000 (so \( t = 0 \) in 2000, \( t = 1 \) is 2100), and \( P(0) \) the number of trolls measured in the year 2000. Suppose the rate of growth is proportional to existing population, say twice the population, and at the beginning of 2000 there were 14,000 trolls. We want to find a formula for \( P(t) \) for all time. Let \( t \) be time, for all times in some range, maybe \( t \in [0,\infty) \), that is, in the period 2000 to 2300. 

**Answer:** a function, not just a number. The equation is: 

\[
P'(t) = 2P(t) \]

(1) This is a start, not a complete solution.

\[
\frac{dL}{dt} = 2L \]

(1000's of trolls/century)

Solve. \( P(t) = 14e^{2t} \). Suppose somebody told you this is the solution.

How do you check? Is it true that \( P(t) = 14e^{2t} \)? Yes.

Is it true that \( P'(t) = 2P(t) \) for all \( t \)? Check: \( P'(t) = 14e^{2t} \). This is indeed equal to \( 2P(t) \) for all \( t \). [It must work for all \( t \); not just for some values.]

Ex. Consider the differential equation: 

\[
y''(t) - y'(t) - 2y(t) = 2e^{2t} - 2. 
\]

We have two proposed solutions: \( y_1(t) = e^{2t} - t \), \( y_2(t) = e^{2t} - t^2 + 1 \).

Are these really solutions?

Let's check \( y_1(t) \). We need \( y_1(t) = 2e^{2t} - 1 \), and \( y_1''(t) = 4e^{2t} \).

So \( y_1''(t) - y_1'(t) - 2y(t) = 4e^{2t} - (2e^{2t} + 2) = 4e^{2t} - 2e^{2t} - 2 = 2e^{2t} - 2 \) (not a solution!)

But for some values of \( t \) (such as \( t = 0 \), \( 2e^{2t} + 2 = 2e^2 + 2 = 2(1) + 2 = 4 \) and not a solution)

\( y_2(t) \). \( y_2(t) = 2e^{2t} - 2t \), \( y_2''(t) = 4e^{2t} - 2 \). So

\[
y_2''(t) - y_2'(t) - 2y_2(t) = 4e^{2t} - 2 - (2e^{2t} - 2t) - 2(2e^{2t} - t^2 + 1) = -2 + 2t - 2 + 2t^2 = 2t^2 + 2t - 4 \not= 0 
\]

Not a solution.

**So \( y_2(t) \) will be a solution. Should say: \( y_1 \) is not a solution \( y_2 \) is a solution.**