Example Consider diff equ. \( y'(t) = -\frac{1}{10} y(t)(4 - y(t)) \) for \( t \in (-\infty, 0) \).

1. To find \( y'(11) \): \( y'(11) = -\frac{1}{10} y(11)(4 - y(11)) = -\frac{1}{10}(4 - 3) = -\frac{3}{10} \).
2. \( y'(11) < 0 \) so \( y \) should be decreasing near \( t = 11 \).

Suppose instead \( y(11) = 6 \). Then \( y'(11) = -\frac{1}{10} y(11)(4 - y(11)) = -\frac{1}{10}(4 - 6) = -\frac{2}{10} \).
So \( y \) should be decreasing near \( t = 11 \).

Back to \( y(11) = 3 \). Approximate \( y(11) \) using the linear approximation.

Also if \( y(11) = 6 \) instead:

New material starts here. Recall linear approximation: for \( h \) small, \( y(t+h) = y(t) + hy'(t) \).
So if \( y(t) = 3 \), then (assuming \( h \) is small enough) \( y(t+h) \approx y(t) + (0.1)y'(t) = 3 + (0.1)(-\frac{3}{10}) = 3.03 \).

Conclusion: \( y(11) \approx 3.03 \).

Suppose instead \( y(11) = 6 \). Then \( y(11.1) \approx y(11) + (0.1)y'(11) = 6 + (0.1)(-\frac{3}{10}) = 5.88 \).

Look at graph above: If \( y(t) \) is in the interval \((0, 4)\), then the function \( y(t) \) is increasing.
For which values of \( y(t) \) do we know that \( y \) is decreasing at \( t \)? If \( y(t) < 0 \), then the diff eqn tells us that \( y'(t) < 0 \), so \( y \) is decreasing.

Consider the function \( y(t) = 4 \) for all \( t \). (Constant function)

Does it solve the eqn? Recall eqn. \( y = \frac{1}{10} y(4 - y) \). For this choice of \( y(t) \), both sides are always zero, so this is a solution. There is one more constant solution: \( y(t) = 0 \) for all \( t \).

Rough picture of graphs of solutions:

Three principles: (1) Solutions curves never cross or merge. (2) For each \( t \) and \( y \), there is a solution satisfying the initial condition \( y(t) = y \). (3) Determine slope of a solution curve just by knowing \( y(t) \) (and fo more complicated cases also \( t \)).

These work for equations saying \( y'(t) \) is some function of \( t \) and \( y(t) \).
These are "direction fields" 

\[
\begin{array}{l}
\text{for } y'(x) = \frac{1}{2} (y(x) - x) \\
\text{for } y'(x) = -\frac{1}{3} x y(x). \\
\text{[check!]}
\end{array}
\]

At the point \((x,y)\), there is a short line segment with slope \(\frac{1}{2} (y-x)\). (For example, along \(y=x\), \(y' = \text{all horizontal, that is, slope 0}\))

So a solution curve (graph of \(y = y(x)\) for some solution) must be tangent to the short line segment drawn at each point that the graph goes through.

These are computer-generated graphs of solution curves on the same direction fields.
Example: Consider \( y(t) = y(t)^4 - 6y(t)^3 + 5y(t)^2 \). (Special: \( t \) does not appear)

Q1: For which numbers \( c = y(t) = c \) (constant) is \( y(t) \) a solution? (Independently, only in \( y(t) \)

Q2: For what values of \( y(t) \) do we know that the solution \( y(t) \) must be increasing (decreasing) at \( t \)?

Let's factor the right-hand side: \( y'(t) = y(t)^2(y(t) - 1)(y(t) - 5) \).

A root for \( y'(t) = 0 \) so look for \( c \) such that \( c^2(c-1)(c-5) = 0 \).

In general form: \( c^2 - 6c^2 + 5c^2 = 0 \) \( \Rightarrow \) values of \( c \) are 0, 1, 5. So the constant solutions are \( y(t) = 0 \) for all \( t \), \( y_2(t) = 1 \) for all \( t \), and \( y_3(t) = 5 \) for all \( t \). This follows.

For Q2: the expression \( y^2(y-1)(y-5) \) is positive if \( y > 5 \)

\(~> 0 \quad ~< 0 \quad ~< 0\)

For \( y < 0 \) negative if \( y \) is in \((1, 5)\).

Positive if \( y \) is in \((0, 1)\) in green.

Positive if \( y < 0 \).

So: \( y(t) \) is increasing whenever \( y(t) > 5 \), \( y(t) \) is in \((0, 1)\), or \( y(t) < 0 \).

\( y(t) \) decreases if \( y(t) \) is in \((1, 5)\).