1. (1 point.) The Squeeze Theorem was invented for the purpose of torturing which of:
   (1) Sequences.
   (2) Calculus students.
   (3) Professors who have to teach it in calculus classes.

2. (18 points.) Consider the power series \[ \sum_{n=1}^{\infty} \frac{(-1)^n(x - 2)^n}{7n^{-3/2} \sqrt{n + 1}} \]. You are told that its radius of convergence is 7. Given this, find its interval of convergence. For any convergence test that you use, be sure to say why it applies.

3. (8 points.) Let \((a_n)_{n=1}^{\infty}\) be a sequence, and let \(L\) be a real number. State the precise definition of what it means to have \(\lim_{n \to \infty} a_n = L\).

4. (12 points.) Define a sequence \((x_n)_{n=1}^{\infty}\) by \(x_n = \frac{n^2}{2n^2 + 1}\) for \(n = 1, 2, \ldots\). For \(\varepsilon = \frac{1}{1000}\), find some integer \(N > 0\) such that for all \(n > N\) we have \(|x_n - \frac{1}{2}| < \varepsilon\), and show that your choice works. (You need not find the best value of \(N\).)

5. (8 points.) Define what it means for a series \(\sum_{n=1}^{\infty} a_n\) to converge to a number \(L\).

6. (15 points.) You want to approximate \(\sum_{n=1}^{\infty} \frac{3}{n^3}\) to within 0.001. Which partial sum should you use? Why? Make sure to explain why the hypotheses of any estimate you use are satisfied.

7. (20 points.) For the differential equation \(y'(t) = (8t^3 - 5)y(t)\), find the recurrence relation for the coefficients of the power series solutions centered at 0.

8. (12 points.) Define a sequence \((a_n)_{n=1}^{\infty}\) by \(a_n = \frac{4 \cos(\sqrt{n})}{\sqrt{n}}\) for \(n = 1, 2, 3, 4, \ldots\). Find its limit (possibly \(\infty\) or \(-\infty\)), giving reasons, or explain why the sequence neither converges nor diverges to \(\infty\) or \(-\infty\).

9. (15 points.) Determine whether or not the series \(\sum_{n=1}^{\infty} (-1)^n \ln \left( \frac{n + 23}{23n + 12} \right)\) is convergent. For any convergence test that you use, be sure to say why it applies.
10. (20 points.) For the differential equation $y''(x) + 3y'(x) - x^2 y(x) = \cos(2x)$, find the terms through degree 4 of the power series solution centered at 0 and satisfying $y(0) = 8$ and $y'(0) = -1$.

11. (16 points.) Determine whether or not the series $\sum_{n=1}^{\infty} \frac{\sin(n(n+1))}{n(n+1)}$ is convergent. For any convergence test that you use, be sure to say why it applies.

12. (12 points.) Define $f(x) = e^{2x^3}$ for all real $x$. Find $f^{(12)}(0)$. (Remember to show your work. Simplify your answer but don’t multiply out powers, factorials, etc.)

13. (16 points.) Find, with justification, some number $n$ such that the approximation of $f(x) = e^x$ by its Taylor polynomial of degree $n$ centered at $\frac{1}{2}$ gives an error of less than $\frac{1}{100}$ on the interval $[0, 1]$. You need not find the smallest choice of $n$.

14. (15 points.) For the differential equation 

$$(x - 9)y''(x) + e^x y'(x) + \frac{y(x)}{x^2 + 4} = 0,$$

give a lower bound for the radius of convergence of power series solutions centered at $x = 3$.

15. (12 points.) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n n! x^n}{(3n)!}$. For any convergence test that you use, be sure to say why it applies.

Extra credit. (Do not attempt these problems until you have done and checked your answer to all the ordinary problems on this exam. They will only be counted if you get 150 points or more on the main part of this exam, and also only if your course grade is B- or better without extra credit.)

EC1. (20 extra credit points.) Find, with justification, a number $N$ such that the $N$-th partial sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ is within $10^{-100}$ of the sum of the series (which is $\frac{1}{e}$). You need not find the best possible choice of $N$.

EC2. (15 extra credit points.) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \sin \left( \frac{1}{2^n} \right) x^n$. For any convergence test that you use, be sure to say why it applies.